EEG/MEG 3:

Time-Frequency Analysis

Olaf Hauk

olaf.hauk@mrc-cbu.cam.ac.uk

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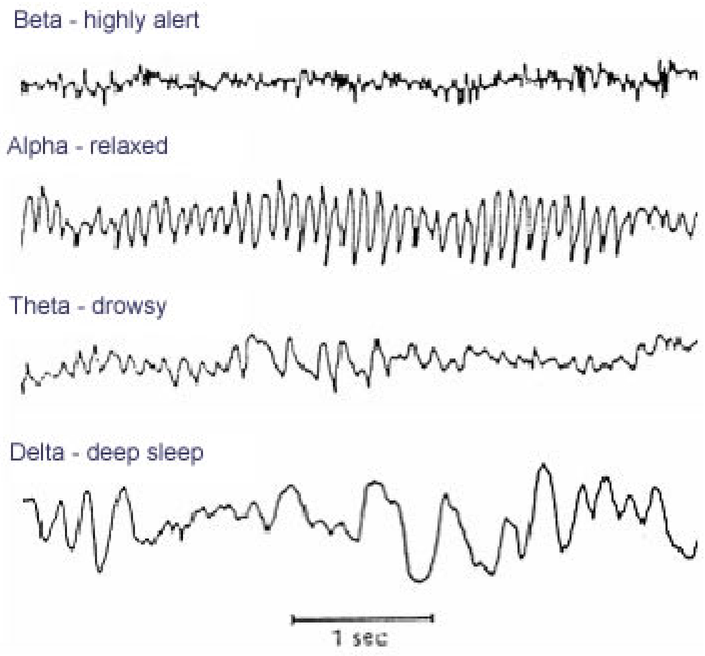
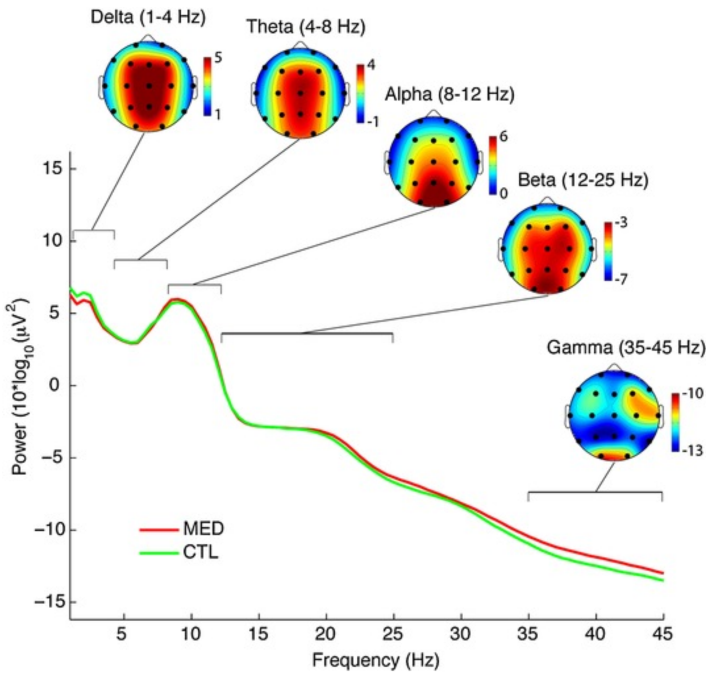
“Brain Rhythms” and “Oscillations”

Time course and topography may differ among different frequency bands

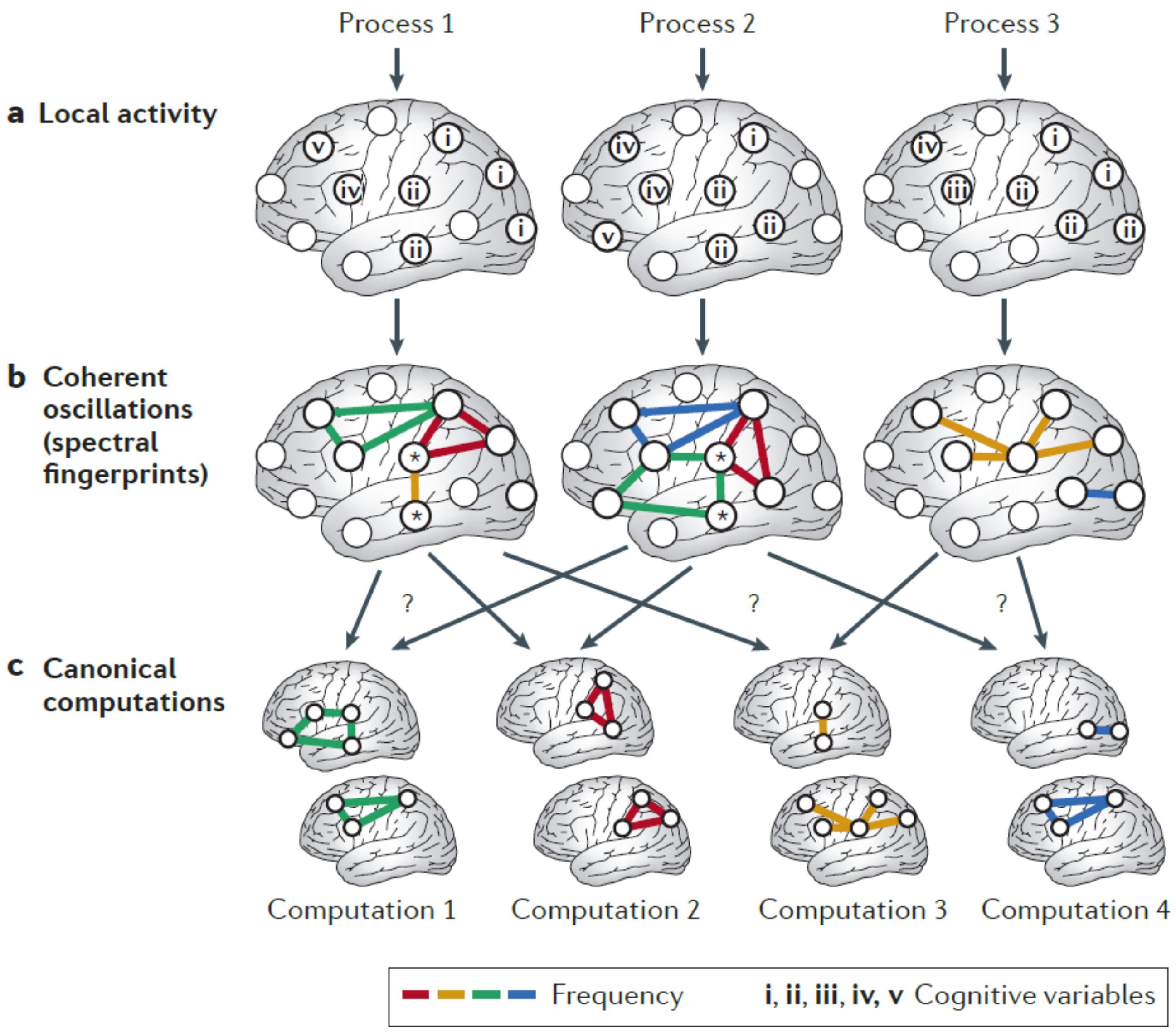
(and may depend on task, environment, subject group etc.)





Cahn et al., Cogn Proc 2010, http://link.springer.com/article/10.1007%2Fs10339-009-0352-1/

“Brain Rhythms” and “Oscillations”



Siegel et al., Nat Nsc 2012

Periodic Signals

A periodic signal repeats itself with a period T.

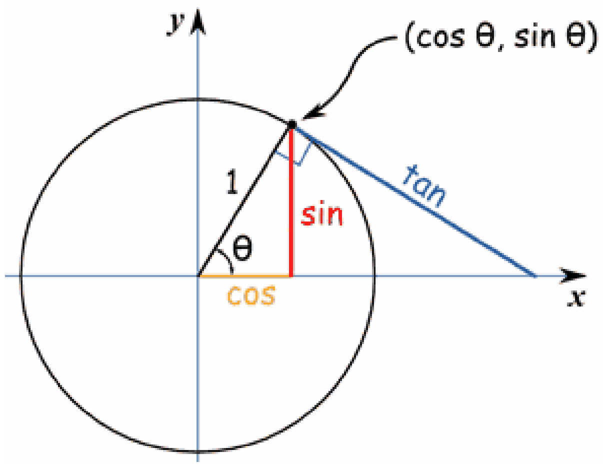
This is the case, for example, for sine and cosine functions:

$$\mathrm{s}(t) = a * sin(2\pi f * t + \theta)$$

a: amplitude

f: frequency

θ : phase

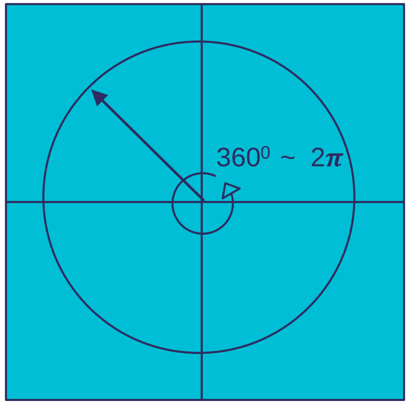


In radians ($\mathbf{2\pi}$ ~ 360 degrees):

$$\boldsymbol{cos(x + 2\pi) = cos(x)}$$
$$\boldsymbol{sin(x + 2\pi) = sin(x)}$$

In degrees :

$$\boldsymbol{cos(x + 360) = cos(x)}$$
$$\boldsymbol{sin(x + 360) = sin(x)}$$



On a unit circle, a 360^0 angle corresponds to a circumference of 2*pi

https://www.youtube.com/watch?v=z82I6u4DFTo

Polar Representation Of Periodic Signals

Euler's Formula

"Complex" numbers can capture the two axes of the coordinate system for the circle around which the vector rotates periodically – this is rather abstract but helps the notation enormously.

$$e^{-i\theta} = \boldsymbol{cos}(\theta) + \boldsymbol{i} * \boldsymbol{sin}(\theta) \quad \boldsymbol{i}\textbf{=}\sqrt{-\mathbf{1}}$$

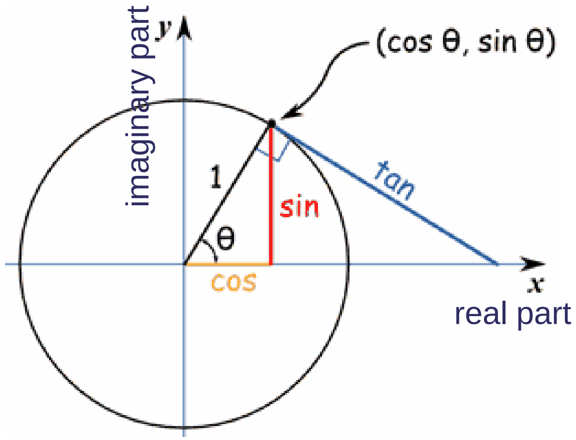
Therefore:

$$cos(\theta) \;=\; real(e^{-i\theta})$$
$$sin(\theta) \;= imag(e^{-i\theta})$$

An oscillation at a particular frequency can be described in a "polar representation":

$$a * e^{-i2\pi f t}$$

a: amplitude
2π: circumference of unit circle
f: frequency
t: time



The Polar Representation Of Periodic Signals

Convenient To Compare Periodic Signals

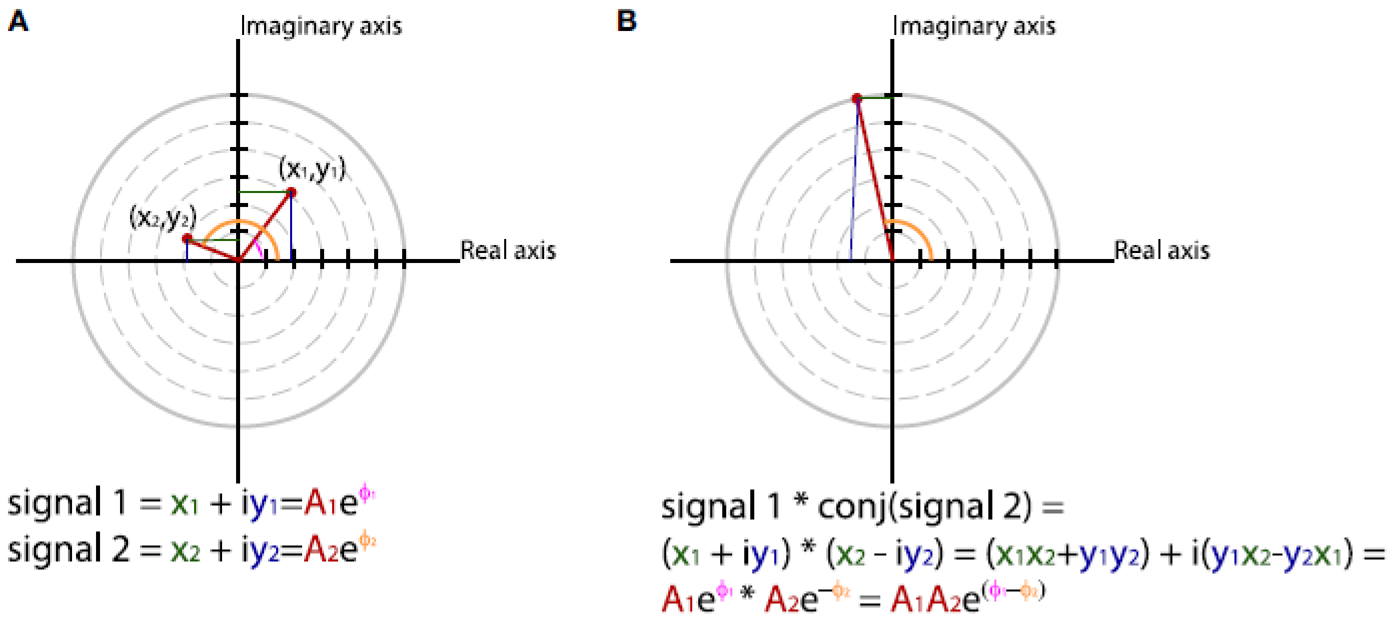
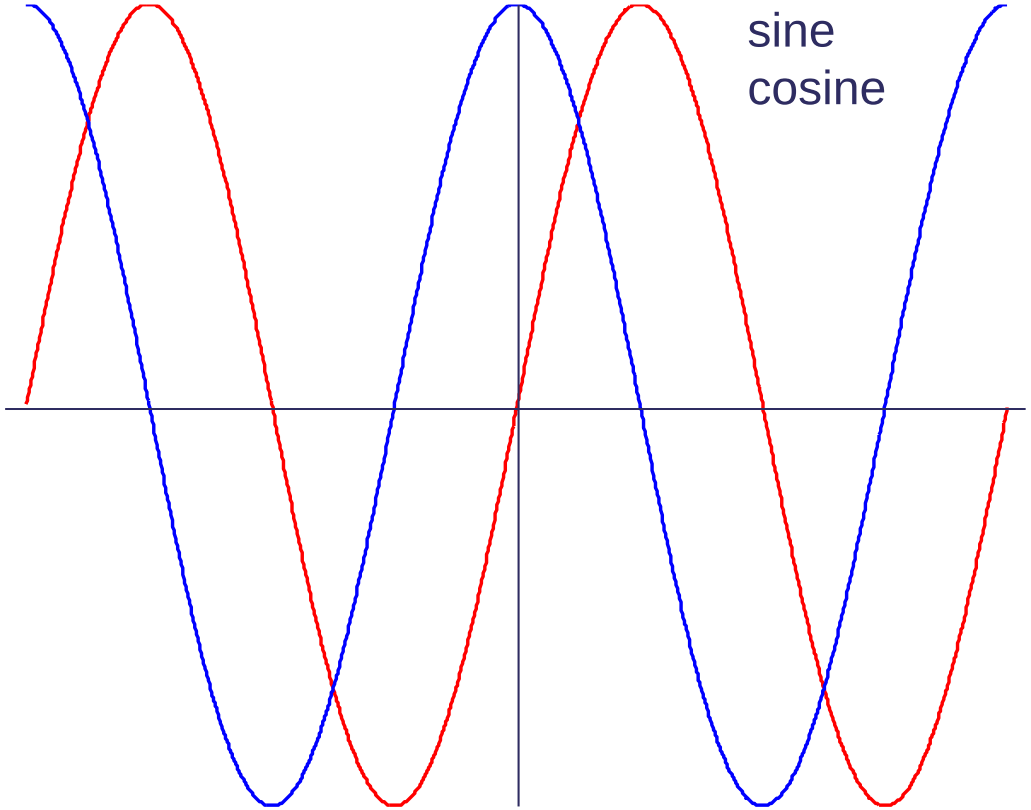


FIGURE 2 | Using polar coordinates and complex numbers to represent signals in the frequency domain. (A) The phase and amplitude of two signals. **(B)** The cross-spectrum between signal 1 and 2, which corresponds to multiplying the amplitudes of the two signals and subtracting their phases.

Bastos & Schoeffelen, Front Syst Nsc 2016
https://www.frontiersin.org/articles/10.3389/fnsys.2015.00175/full

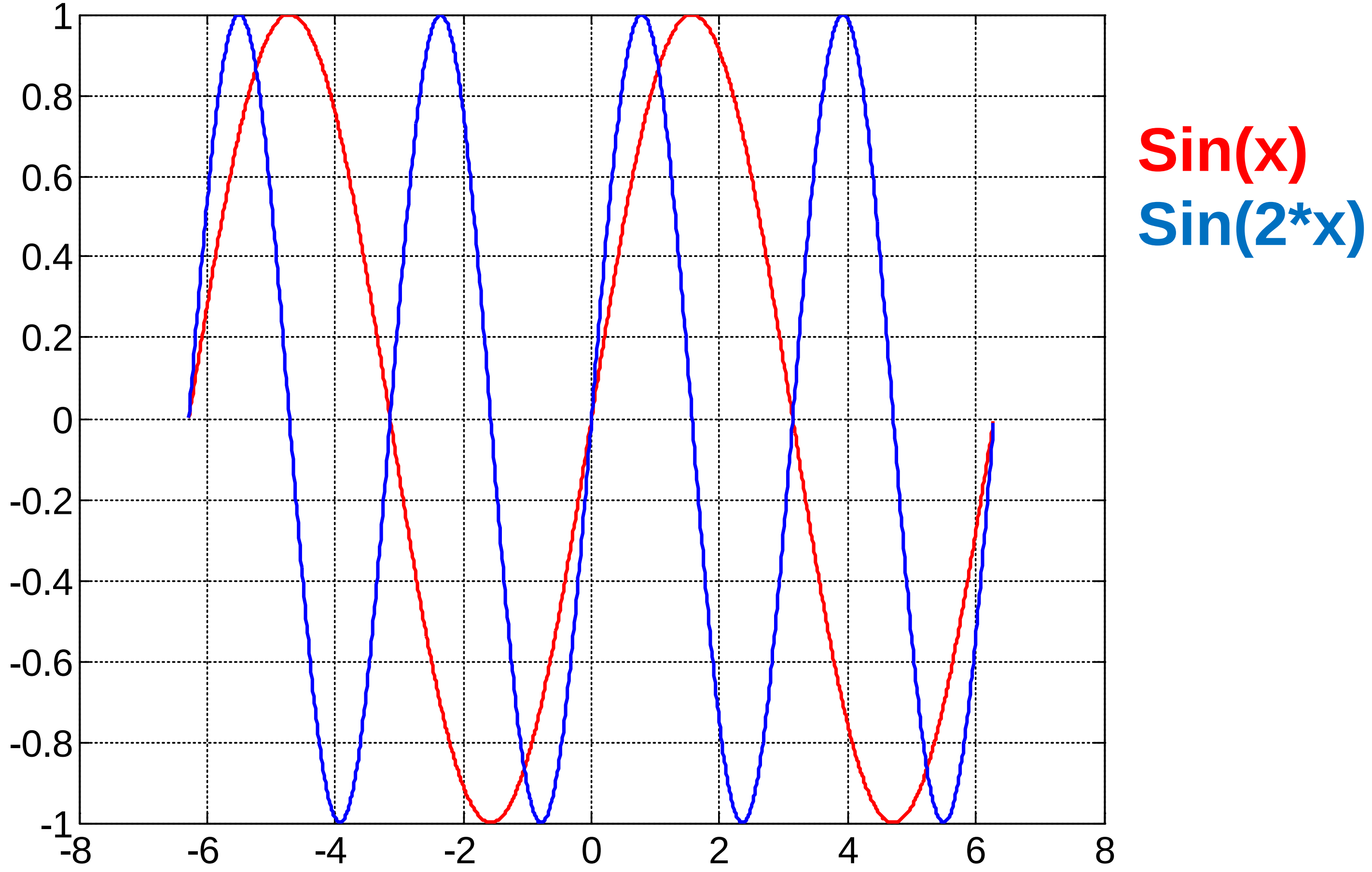
Sine and Cosine Are Orthogonal to Each Other

(at a given frequency)



$$\int \sin(f * x)\, cos(f * x) dx = 0$$

Sine/Cosine At Integer Frequency Intervals Are Orthogonal



$\int \sin(m * f * x)\, sin(n * f * x) dx = 0$ for integer m, n

Entering the Frequency Domain:

Fourier Transform in Words

What you want:

You've got a signal consisting of N sample points (equidistant).
You want to know which frequencies contribute to the signal, and how much.

In other words:
You want to describe your signal as a linear combination of sines and cosines,
ideally of orthogonal basis functions made up of sines and cosines.

What you've got:

With N samples, you can estimate at most N independent parameters.

You cannot estimate frequencies above half of the sampling frequency SF (Nyquist).

For a given frequency, sine and cosine are orthogonal,
i.e. 2 basis functions per frequency.

Entering the Frequency Domain:
Fourier Transform in Words

Divide the frequency range 0 to SF/2 evenly into N/2 frequencies.

For every frequency, create a sine and a cosine.

Use these (orthogonal) sines and cosines as your basis functions.

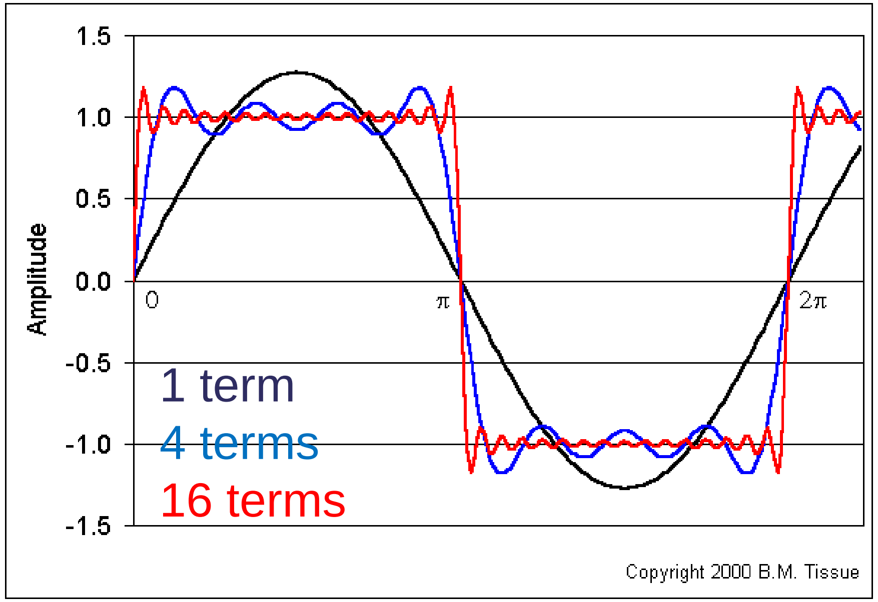
Project these basis functions onto your data, get the amplitudes for individual basis functions – that is your frequency spectrum.

Fast Fourier Transform (FFT): A fast algorithm to do this.

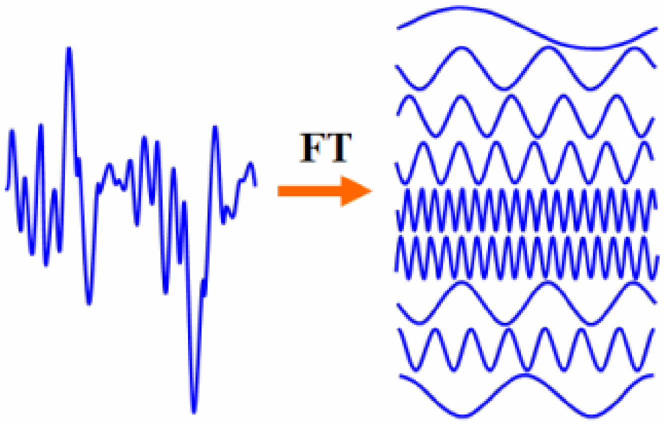
(I’m cheating a bit, assuming an appropriate N and ignoring the mean. But the principle is ok.)

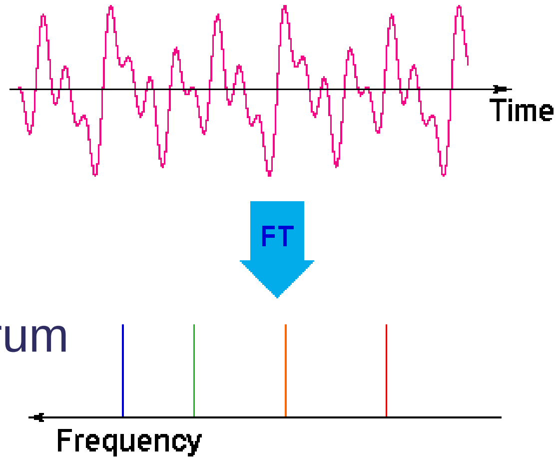
The Fourier (De-)Composition

Approximating a step function
with Fourier terms



Decomposing signals
into sine/cosine terms



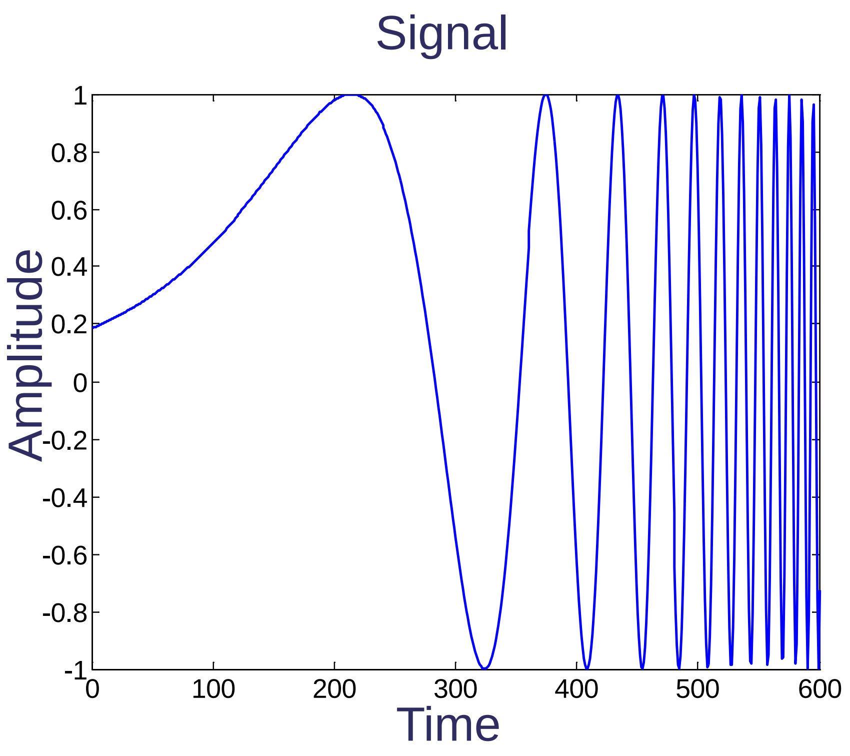


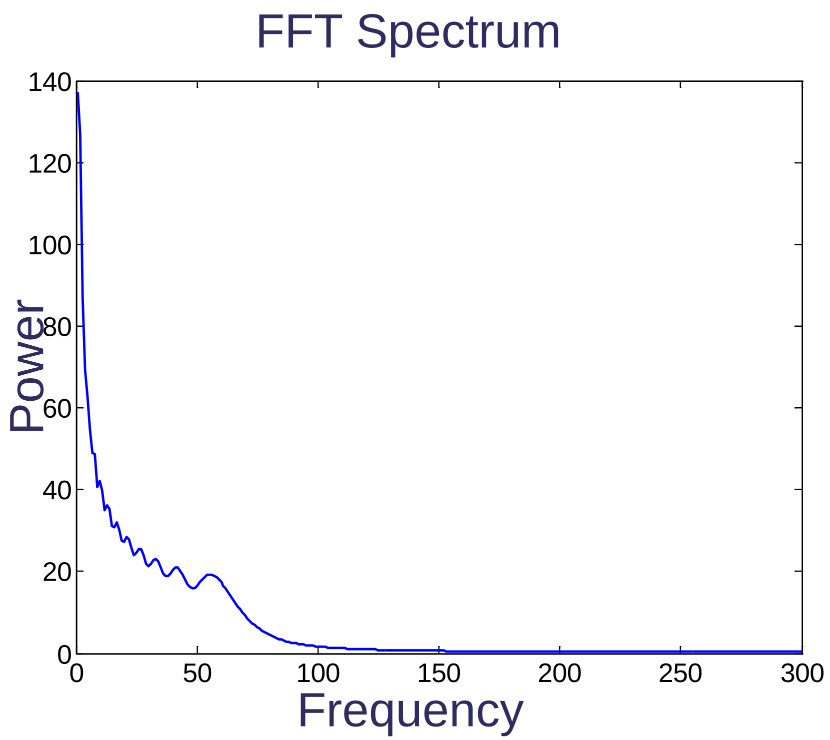
Frequency Spectrum

Motivation for Time-Frequency Analysis

Fourier Transform assumes sines and cosines with constant amplitudes across the whole time series ("stationarity").

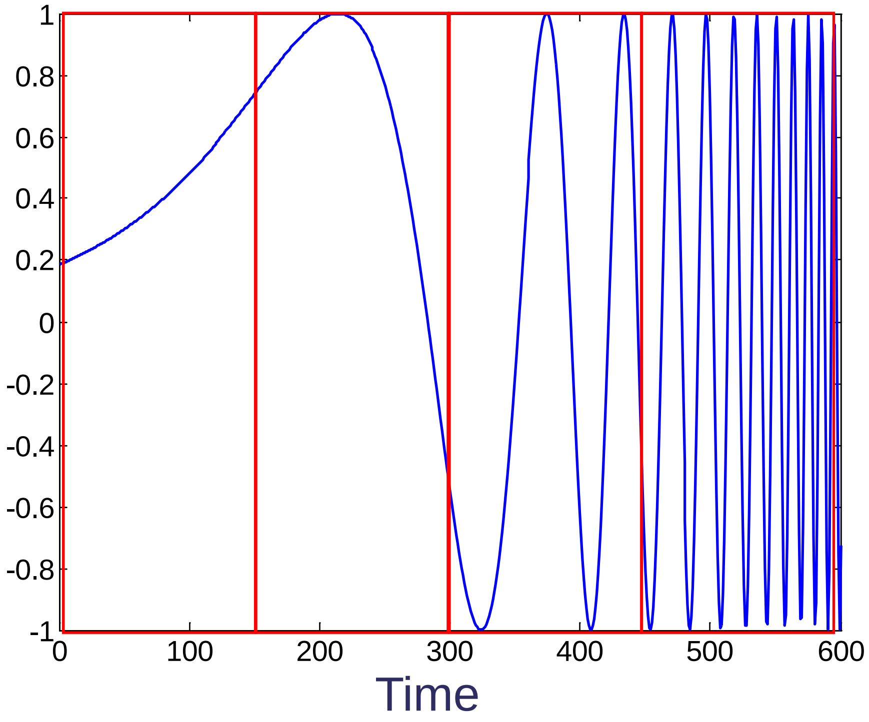
But what does an FFT mean for a signal like this?





Motivation for Time-Frequency Analysis

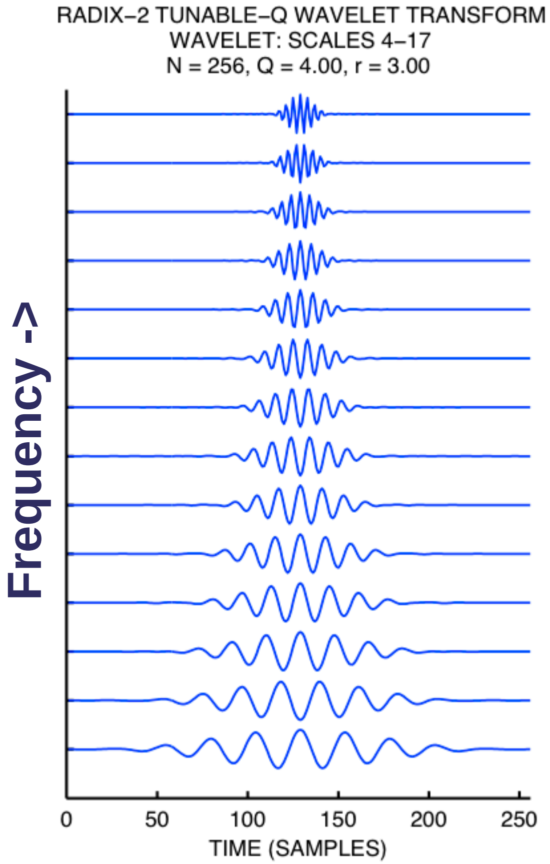
You could run separate FFTs for different (sliding) time windows:



But different window sizes are more or less optimal for different frequencies.
Run different FFTs with different window sizes for different frequency ranges? Ouff.

Time-Frequency Analysis: Wavelets ("little waves")

Wavelets provide an optimal trade-off between frequency and time resolution.



Wavelets are getting "broader" with decreasing frequency

=>

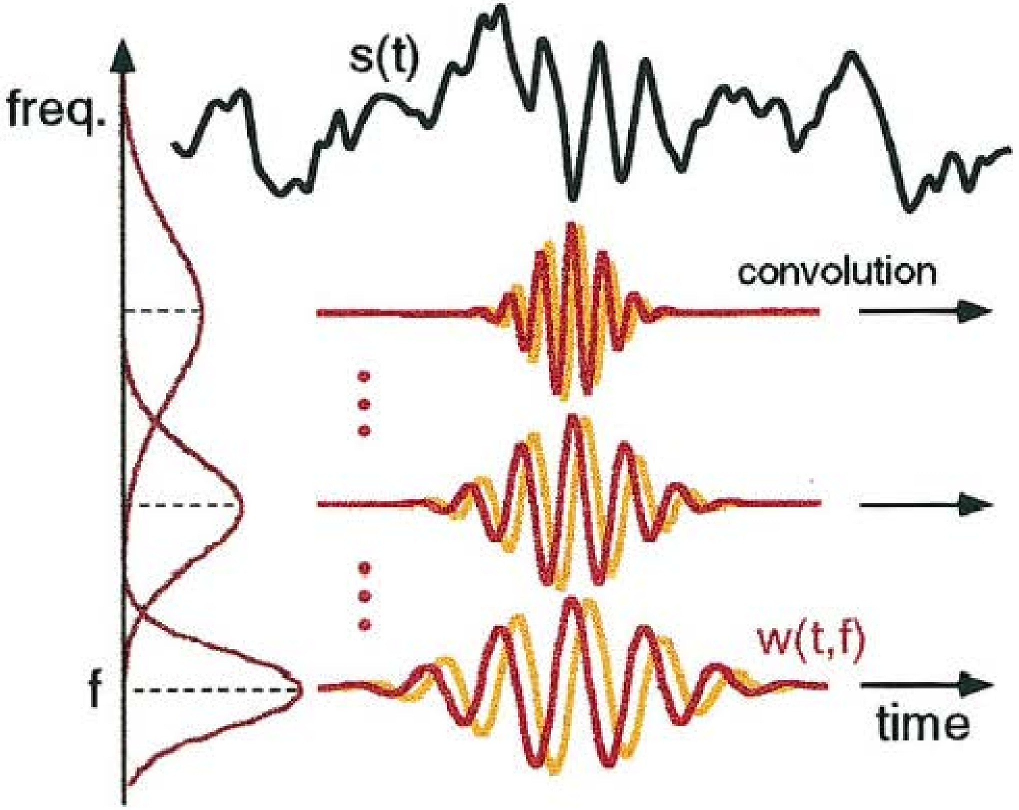
Time resolution decreases as frequency decreases

Wavelets are convolved with the data to give instantaneous amplitude and phase estimates for different frequency ranges.

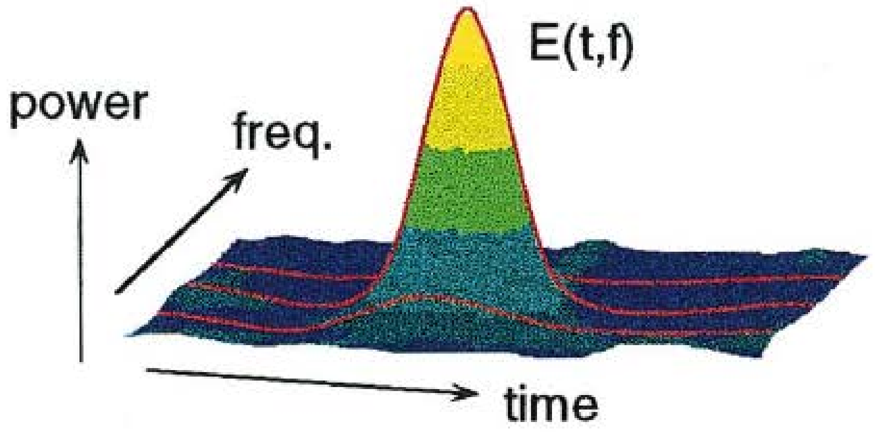
Time-Frequency Analysis: Wavelets

Wavelet Transform

Trade-off between time and frequency resolution



Time-Frequency Power



Tallon-Baudry & Bertrand, TICS 1999
https://pubmed.ncbi.nlm.nih.gov/10322469/

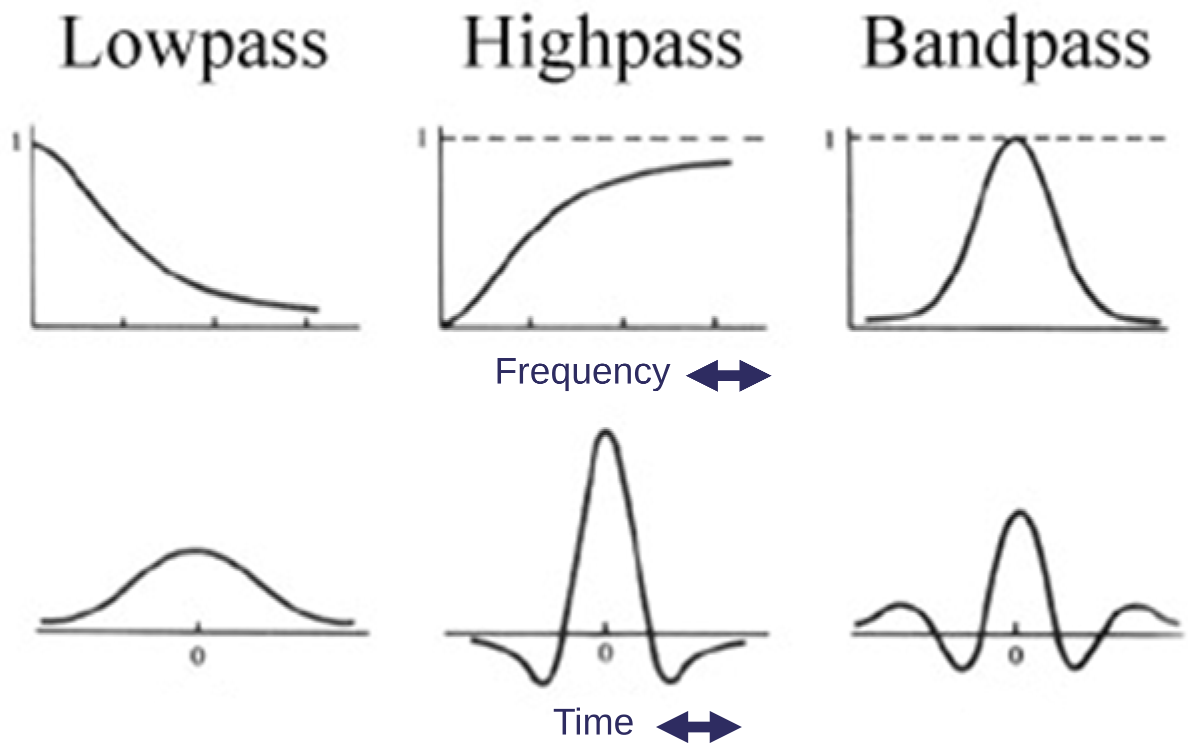
Basic Principals of Frequency Filtering

Time-domain and frequency-domain filtering are two sides of the same coin:

One type of frequency-domain filtering corresponds to one type of time-domain filtering.

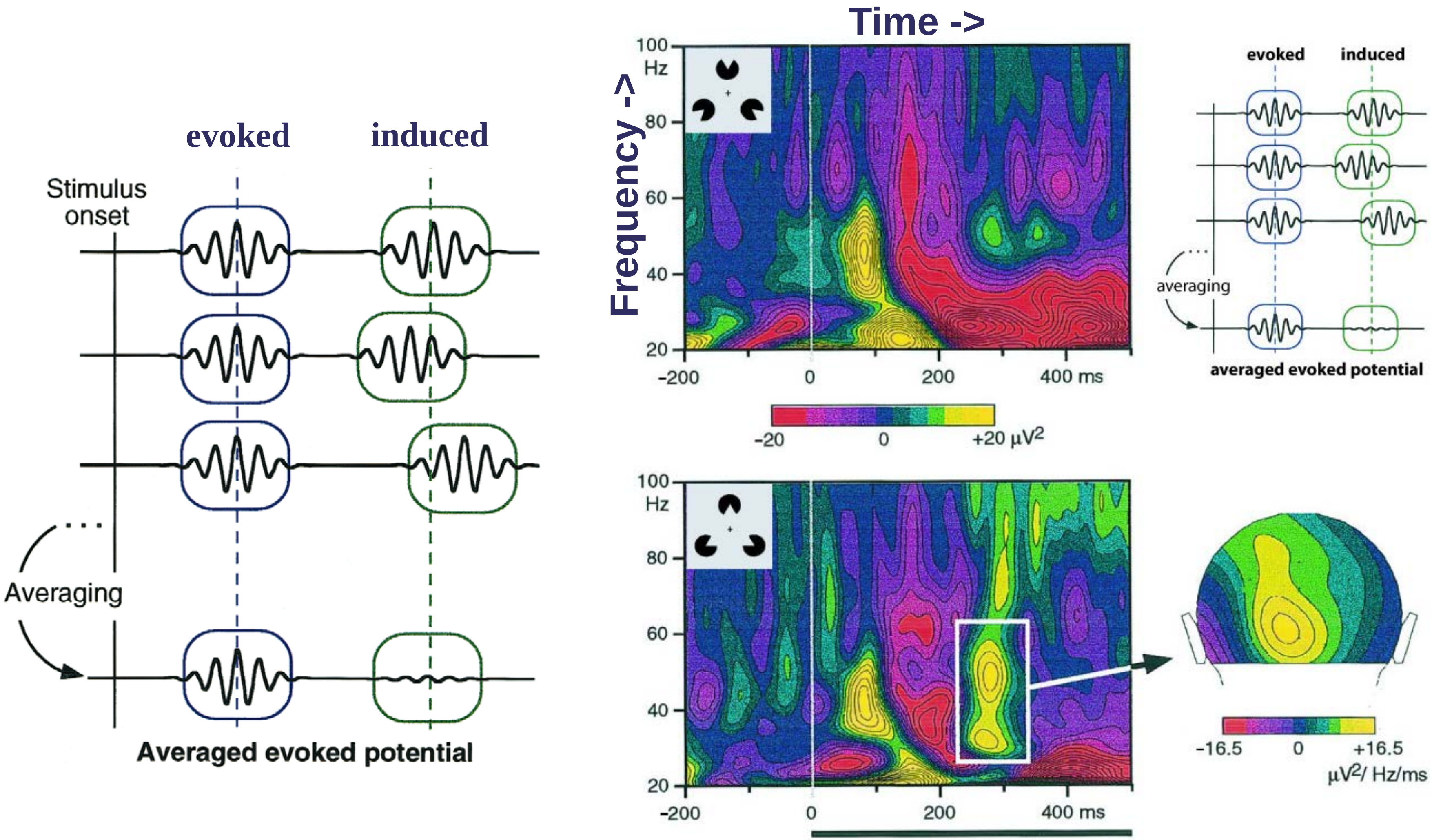
Frequency domain

Time domain



https://uk.mathworks.com/matlabcentral/fileexchange/51155-time-domain-filtering-vs-frequency-domain-filtering-in-images

Evoked and Induced Rhythmic Activity

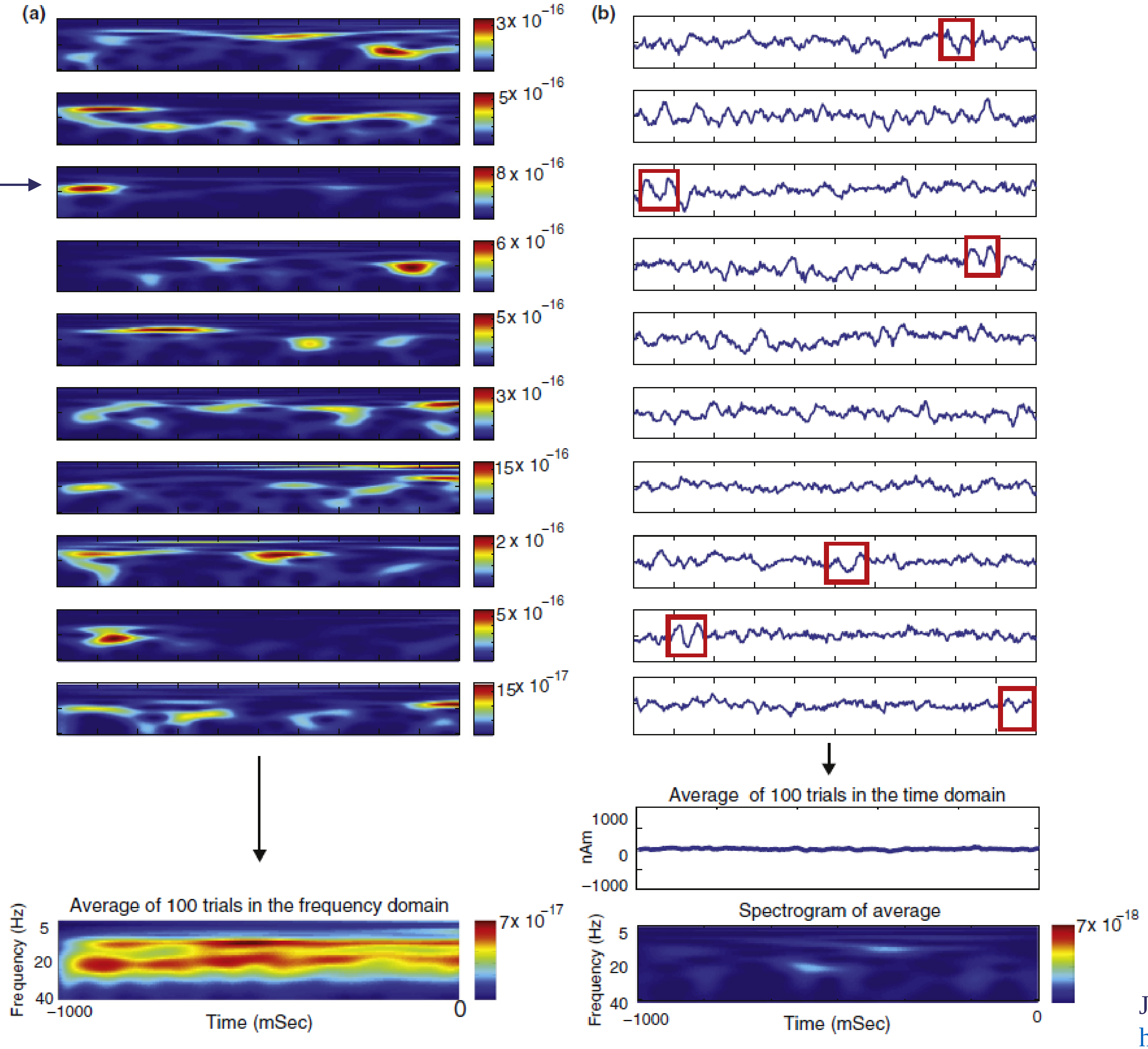


Tallon-Baudry & Bertrand, TICS 1999
https://pubmed.ncbi.nlm.nih.gov/10322469/

When brain rhythms aren't "rhythmic" – the example of beta "oscillations"

"beta bursts"

rather than "oscillations"



Jones et al., Curr Op Neurobiol 2016
https://pubmed.ncbi.nlm.nih.gov/27400290/

A Very Rough Rule of Thumb

One needs at least 2 cycles of a frequency to get a meaningful estimate (of amplitude, phase, etc.)

Duration (in ms) of 2 cycles at frequency f (in Hz): 2*1000/f

1 Hz: 2000 ms = 2 s

10 Hz: 200 ms = 1/5 s

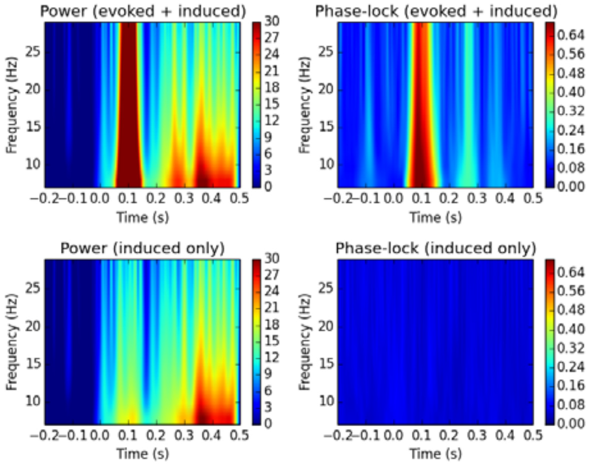
40 Hz: 50 ms = 1/20 s

100 Hz: 20 ms = 1/50 s

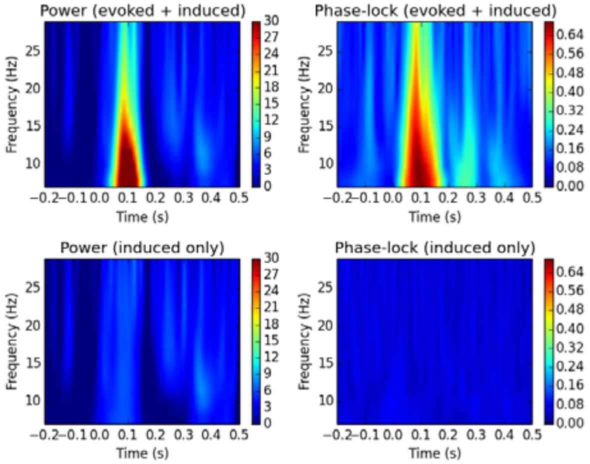
The lower the frequency, the longer the time window required to estimate the signal.

Effect of Number of Cycles

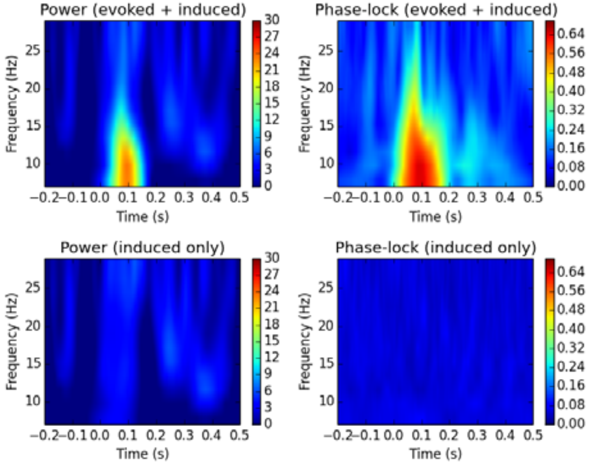
1 cycle



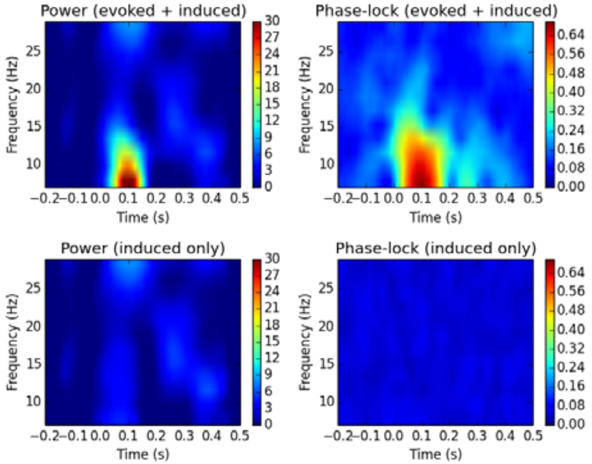
2 cycles



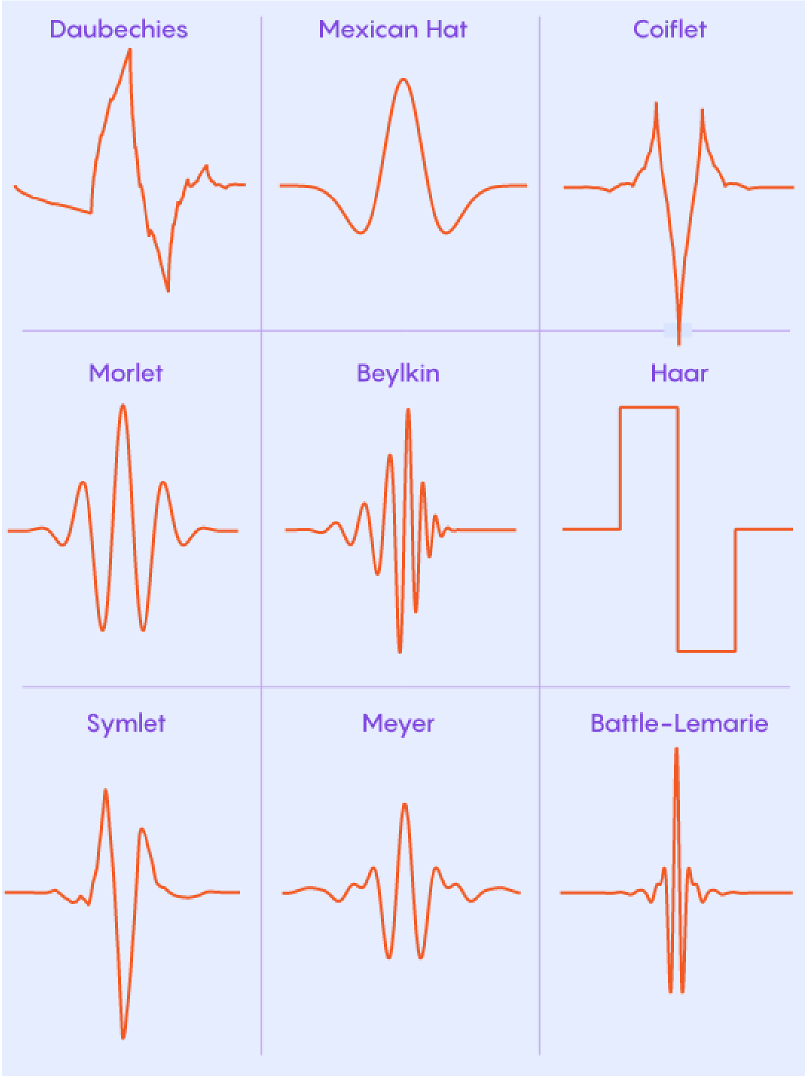
3 cycles



Freq/3 cycles



The Wavelet Zoo



https://www.quantamagazine.org/how-wavelets-allow-researchers-to-transform-and-understand-data-20211013/

“Single-Trial Analysis” and Source Estimation

Computing the power of a signal is a non-linear transformation.

Linear transformations are associative:

$$T(a+b) = T(a)+T(b)$$

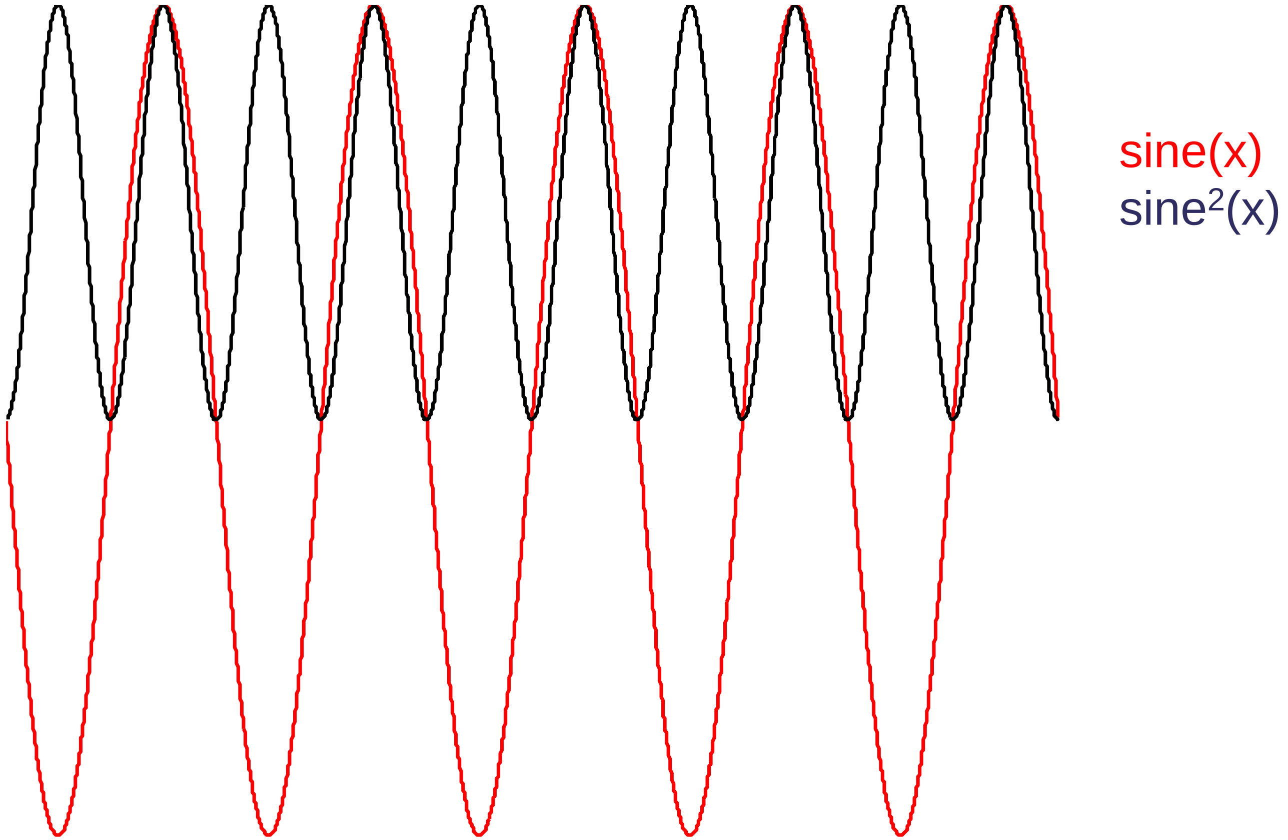
Therefore, the result is the same whether you apply a linear transformation before or after averaging your epochs.

Spectral power is non-linear!
If you want the average power, you have to compute power for individual epochs first, then average.

The noise level and a priori knowledge about sources will be very different for single trials compared to the average.

For example, a single/multiple dipole model may be justified for the average (e.g. auditory P1 etc.), but not for single trials.

Power Estimation Changes the Time Course



For example, the frequency spectrum for sine(x) and sine^2(x) are very different.

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Thank you