Identifying harmonics in magnetoencephalographic data: an EMD perspective

Marco Fabus[1, 3], Mark Woolrich[2, 3], Katie Warnaby[1, 3], Andrew Quinn[2, 3]

[1]Oxford Centre for Functional MRI of the Brain (FMRIB), Nuffield Department of Clinical Neurosciences, University of Oxford, Oxford, UK
[2]Oxford Centre for Human Brain Activity (OHBA), Department of Psychiatry, University of Oxford, Oxford, UK
[3]Wellcome Centre for Integrative Neuroimaging (WIN), University of Oxford, Oxford, UK

Correspondence:
marco.fabus@ndcn.ox.ac.uk

Introduction

Neurophysiological recordings often show non-sinusoidal features (1). Such waveforms have harmonics in their spectra. Harmonics can produce spurious results in connectivity analysis (2). Here we ask the question: *what exactly is a harmonic?*

Most literature classes signals as harmonics if they have an integer frequency ratios. However, independent oscillations can easily satisfy this. We propose a rigorous definition of when signals are harmonically related using instantaneous frequency. Exploring the mathematical structure, two types of harmonics emerge. We link the results to Empirical Mode Decomposition (EMD) and show an example MEG motor-mu application.

MEG Methods

- Randomly chosen resting-state recording from the CamCAN project (3, 4).
- 562s of single gradiometer sensor-space data over the midline motor cortex down-sampled to 400Hz.
- Pre-processing: 0.1-125Hz bandpass and 48-52Hz/98-102Hz notch filters, ICA clean-up using correlation with EOG and ECG channels.
- EMD: Masked EMD (5), maximum number of Intrinsic Mode Functions (IMFs) = 5, masks = [60, 25, 10, 5, 2]Hz
- For summary statistics, data split into 20 segments, mean ± standard deviation presented
- Cycles identified from jumps in instantaneous phase, only those with IF ≥ 0 and IA in 50th percentile kept.

Theory

Harmonic Conditions

A joint sum of sinusoids $x(t) = \sum_{n=1}^{N} a_n \cos(\omega_n t + \phi_n)$ is a harmonic structure if and only if:

1. The joint function x is periodic with the same period as the base, i.e. $x(t+T) = x(t)$,
2. The joint instantaneous frequency f_J is well-defined, i.e. $f_J \geq 0$ for all t.

- *Intuition:* harmonics should only change fine details of waveform shape, the lowest frequency base determines "most" properties.
- First condition equivalent to integer frequency ratios and a constant phase relationship.
- Second condition ensures no new prominent extrema (Figure 1).
- Joint IF for $x(t)$: $$2\pi f(t) = \frac{\sum_{n,m} [a_n a_m \omega_m \cos((\omega_n - \omega_m)t + (\phi_n - \phi_m))]}{(\sum_n a_n \cos \omega_n t)^2 + (\sum_n a_n \sin \omega_n t)^2}$$
- Case N=2: condition #2 simplifies to $af \leq 1$, where a and f are the frequency and amplitude ratios of signals.
- Link to EMD: $af=1$ determines extrema rate and signal splitting into one or two IMFs ((6), Figure 2). => framework can help recombine split modes.
- Case N=∞: IF only well-defined if harmonics amplitude falls faster than $1/n^2$ => two types of harmonics (weak and strong, Figure 3).
- Links to amplitude modulation and the Riemann Zeta Function still being explored.

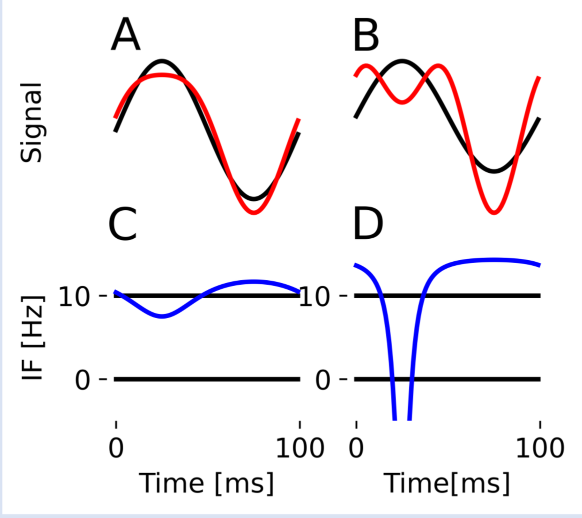

Figure 1: Harmonic intuitions. Plots show sum of a 10Hz and 20Hz waves and their IF. (A, C) 20Hz amplitude is low (harmonic), IF is well-defined. (B, D), 20Hz amplitude is high (not a harmonic), IF negative.

Instantaneous Frequency (IF)

We define IF using the analytic signal. For a real signal $u(t)$, define its analytic signal $x_A = u(t) + iv(t)$, where $v(t)$ is the Hilbert transform:

$$v(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{u(\tau)}{t - \tau} \mathrm{d}\tau$$

Rewriting x_A in polar form, we define IF as

$$f(t) = \frac{1}{2\pi} \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{1}{2\pi} \frac{u\dot{v} - v\dot{u}}{u^2 + v^2}$$

where dots are time derivatives.

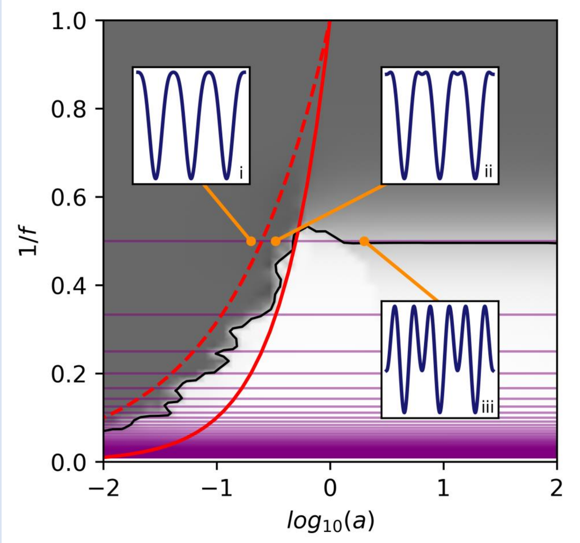

Figure 2: EMD separation of two tones in relation to harmonics. Gray shading shows EMD separation (6). Purple lines show harmonic condition 1. Red lines $af = 1$ (solid) and $af^2 = 1$ (dashed). Insets show (i) A strong harmonic structure, (ii) A weak harmonic structure, (iii) independent oscillations.

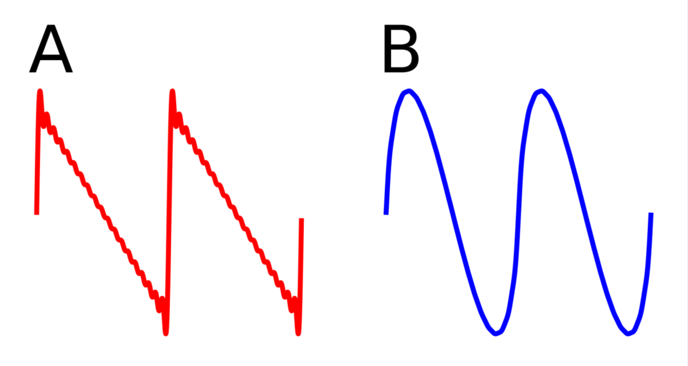

Figure 3: Weak / strong harmonics structures. (A) Weak structure, secondary extrema present and IF not defined in N=∞ limit. (B) Strong structure, no secondary extrema and IF always well-defined.

MEG Application

- Single-subject non-sinusoidal motor mu.
- Due to limited IMF bandwidth, masked EMD splits signal and harmonic (IMF-2 and IMF-3, Figure 4).
- Adding IMFs where harmonic conditions are met reconstructs non-sinusoidal shape (fast rising edge).
- IF ratio = 2.2 ± 0.16 across segmented recording.
- Further theoretical work and work on automatising shape reconstruction ongoing.

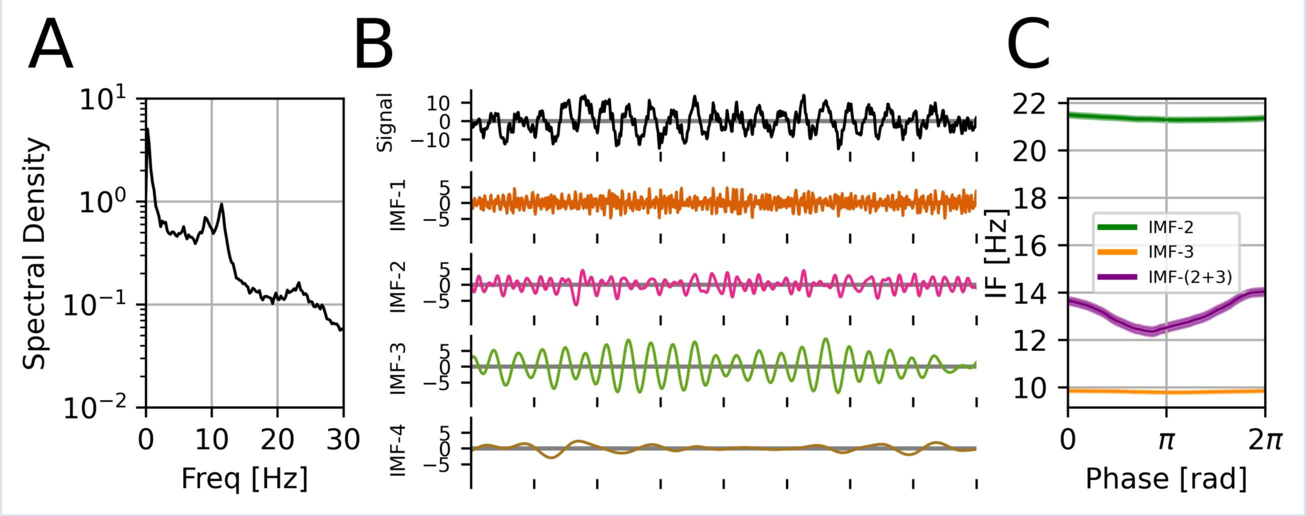

Figure 4: Application to motor MEG. (A) Power spectral density of the data. A base around 11Hz and a harmonic around 22Hz are present. (B) Example 2s of masked EMD. Base is in IMF-3 and harmonic is in IMF-2. (C) Phase-aligned IF (mean ± SEM across cycles). Both IMF-2 and IMF-3 are nearly sinusoidal. Adding IMFs reconstructs the non-sinusoidal shape (purple).

References

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