Functional Connectivity

Rik Henson

MRC CBU, Cambridge

Structural, functional & effective connectivity

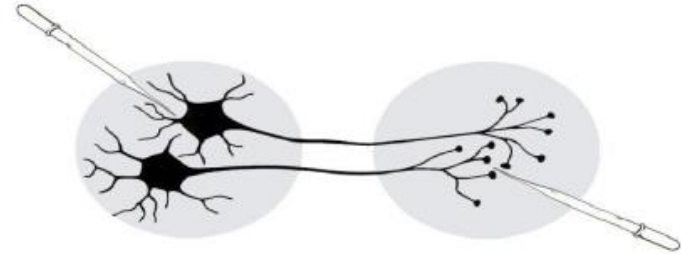structural connectivity   functional connectivity   effective connectivity
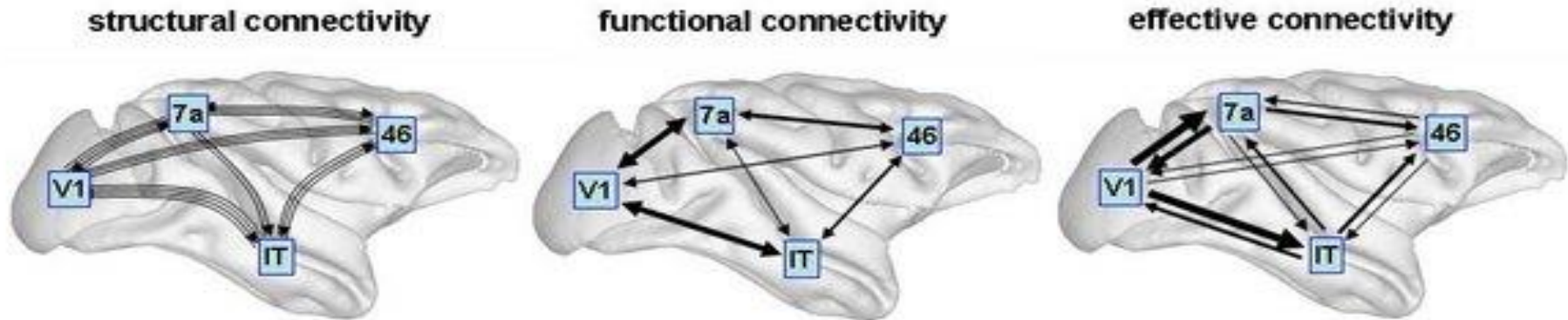
- **Structural/anatomical connectivity**

  = presence of axonal connections / white matter tracks (eg, DWI)

- **Functional connectivity**

  = statistical dependencies between regional time series (eg, ICA)

- **Effective connectivity**

  = causal (directed) influences between neuronal populations (eg, DCM)

    (based on explicit network models)
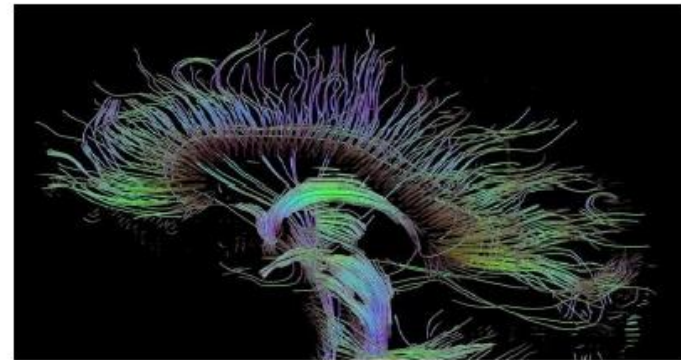
Structural vs Functional connectivity
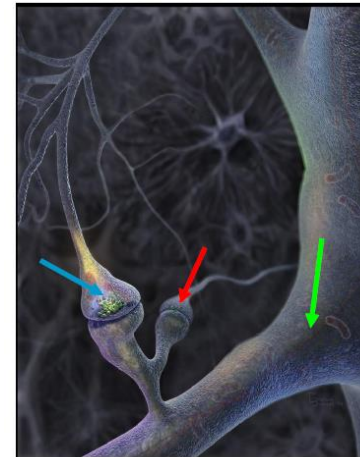
- Tracing studies

- Tractography from DWI

But functionally, effect of one neuron on another can depend on:

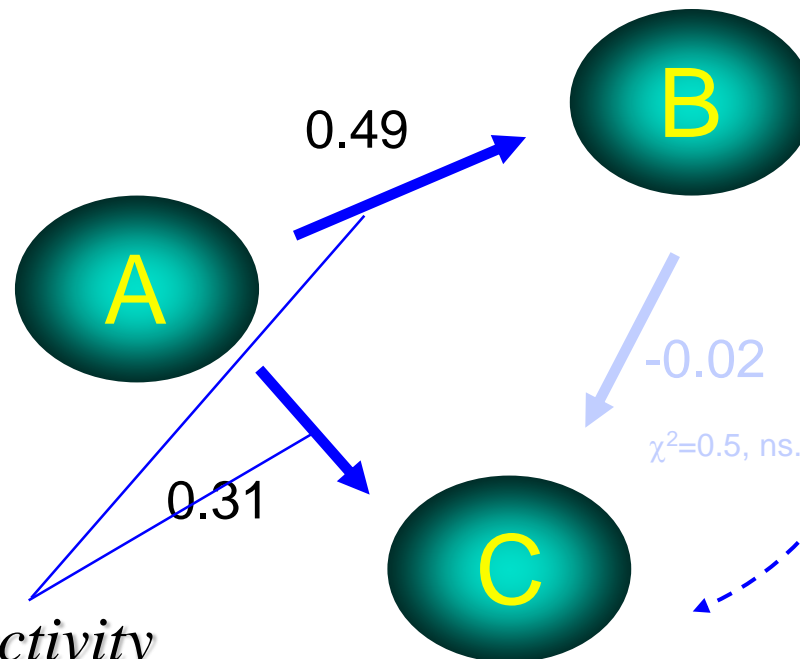- – Activity of a third (gating)

- – Rapid changes in plasticity

Functional vs Effective connectivity

No connection between B and C, yet B and C correlated because of common input from A, eg:

A = V1 fMRI time-series
B = 0.5 * A + e1
C = 0.3 * A + e2

Correlations:

	A	**B**	**C**
	1		
	0.49	**1**	
	0.30	**0.12**	**1**

0.49

0.31

B

A

C

-0.02

χ^2=0.5, ns.

Functional connectivity

Effective connectivity

Functional/Effective Connectivity for fMRI

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Functional connectivity

- Useful when no model, no experimental perturbation (eg resting state)

- Popular examples: seed-voxel correlations, PCA, ICA, etc

- Graph-theory summaries of functional networks

- Correlations in fMRI timeseries could be spurious haemodynamics (e.g, effects of heart-rate/breathing; movement confounds...)

- Condition-dependent changes in functional connectivity (e.g, PPIs...)

Effective-connectivity: Definitions of Causality?

1. Direct experimental interventions (e.g, lesion, drugs)

2. Indirect experimental manipulations (e.g, PPI, DCM)

3. Network model inference (e.g, SEM, DCM)
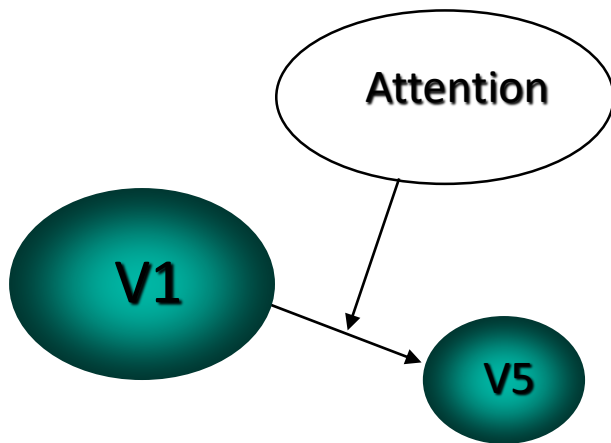
4. Temporal precedence (e.g, Granger Causality, DCM)

5. …

Effective-connectivity: Definitions of Causality?

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5. …

2. Condition-dependent changes: eg PPI
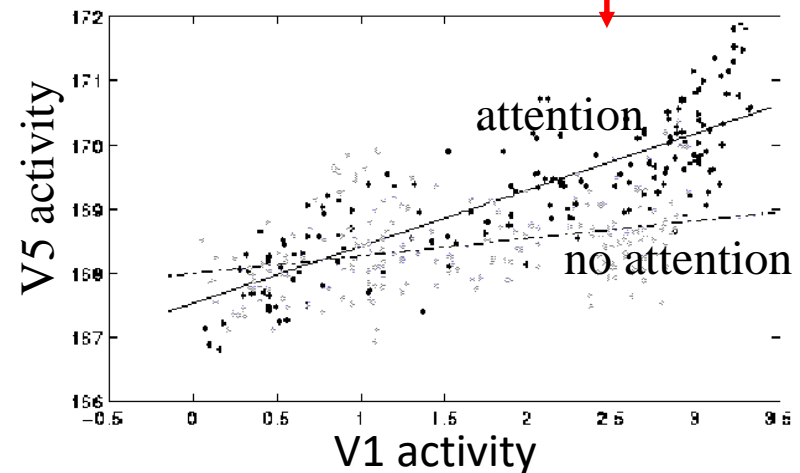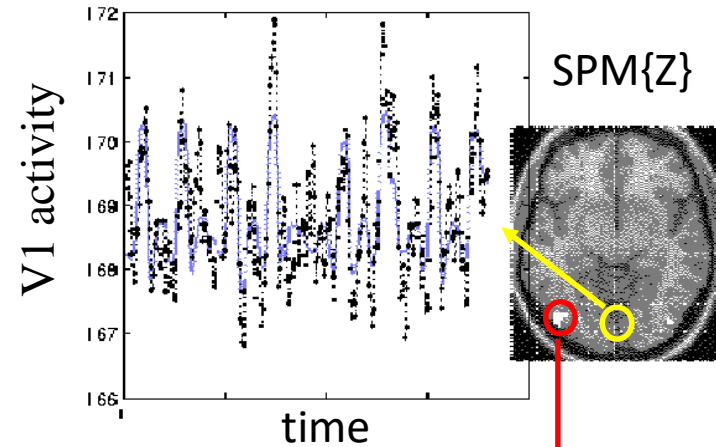
Parametric, factorial design, in which one factor is psychological (eg attention)

...and other is physiological *(viz. activity extracted from a brain region of interest)*

SPM{Z}
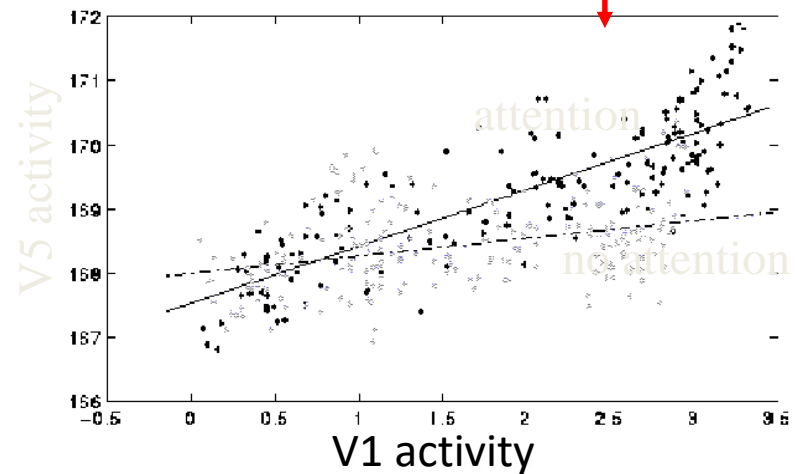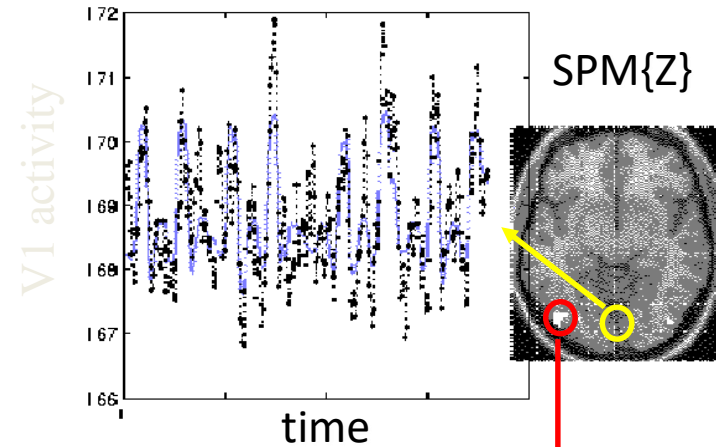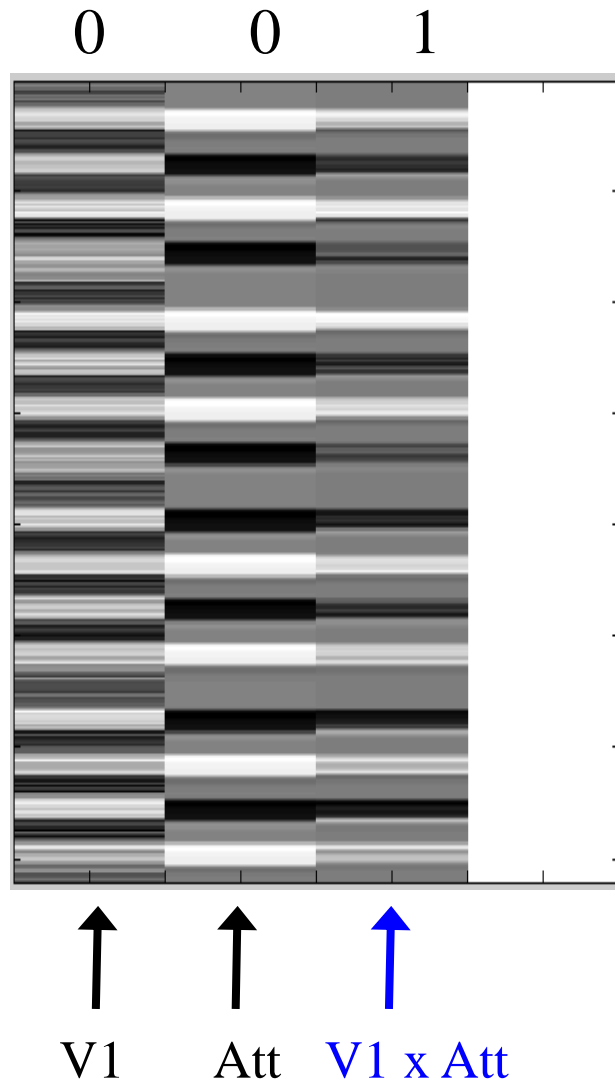
Attention

V1

V5

Attentional modulation of V1 - V5 connectivity

2. Condition-dependent changes: eg PPI

0　　0　　1

V1　　Att　　V1 x Att

SPM{Z}

V1 activity

time

V5 activity

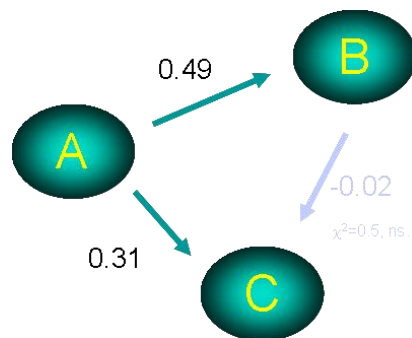attention

no attention

V1 activity

Effective-connectivity: Definitions of Causality?

1. Direct experimental interventions (e.g, lesion, drugs)

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5. …

3. Explicit Network Models of Causality

- (Bivariate) correlations do not use an explicit network (graph) model

- Structural Equation Modelling (SEM) can test different network models, by simply comparing *predicted* with *observed* covariance matrices, but…

  - has no dynamical model (stationary covariances)
  - has no neural-BOLD model
  - cannot test some graphs, eg loops (no temporal definition of direction)
  - restricted to classical inference comparing nested models

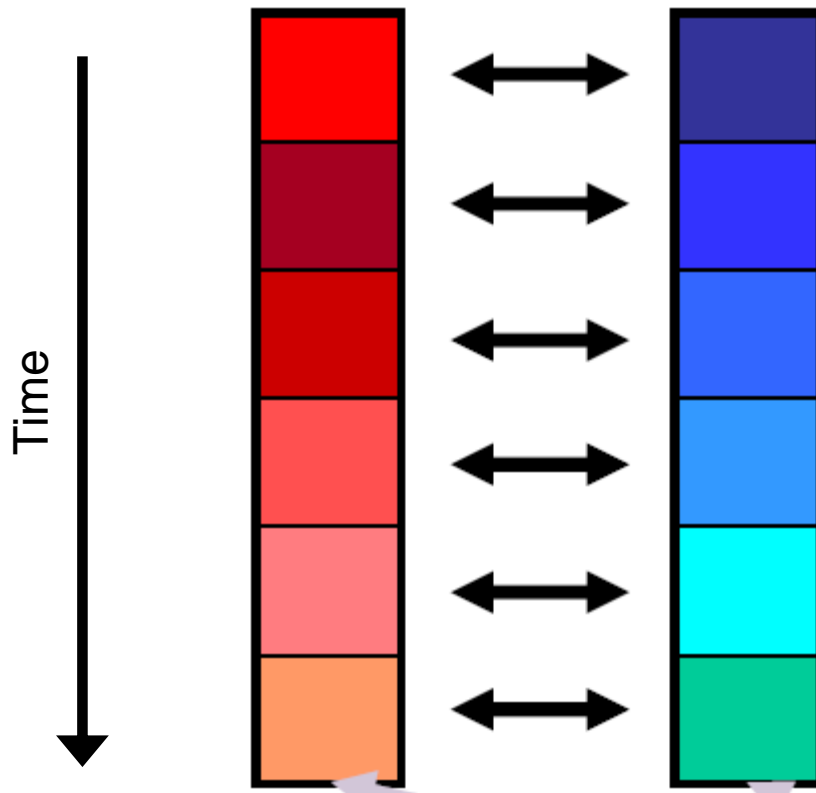# Effective-connectivity: Definitions of Causality?

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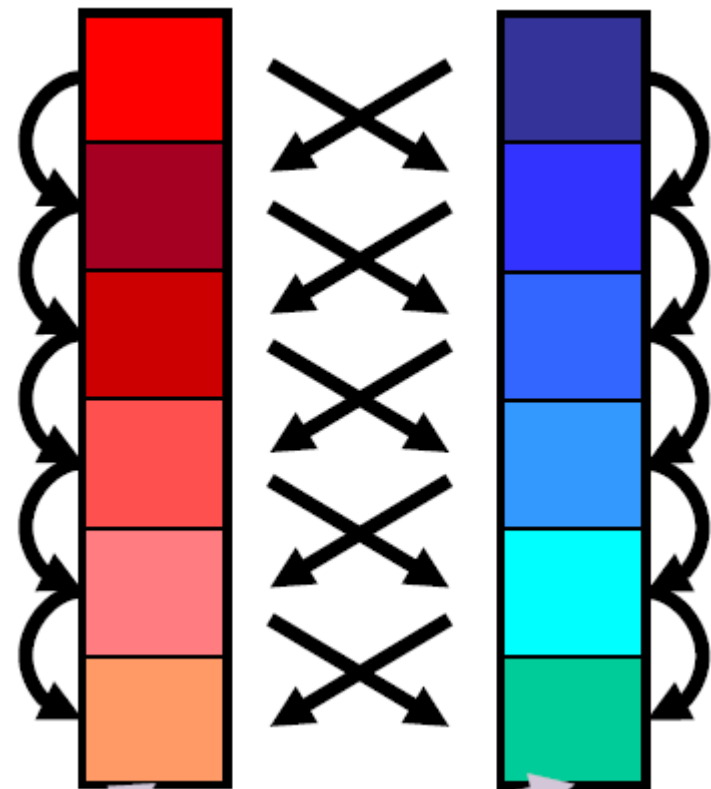
5. …

4. Temporal definition of Causality

Stationary
(correlations, SEM)

Dynamic
(Granger, DCM)

Time

("unfolding" in time is one way to infer direction of connectivity)

4. Note on temporal causality and fMRI

- Problem with time-based measures of connectivity arises with fMRI: BOLD timeseries is not direct reflection of Neural timeseries

    - (e.g, peak BOLD response in motor cortex can precede that in visual cortex in a visually-cued motor task, owing to different neural-BOLD mappings)

- This compromises methods like Granger Causality and Multivariate Auto-Regressive models (MAR) that operate directly on fMRI data
    (Friston, 2010; Smith et al, 2011)

- Note that this does not preclude these methods (eg MAR) for MEG/EEG timeseries, assuming these are more direct measures of neural activity

=> Development of DCM

1. *Dynamic:* based on first-order differential equations

    – at level of neural activity, with separate haemodynamic model for fMRI

2. *Causal:* based on explicit directed graph models

3. *Modelling:* designed to test experimental manipulations

    – "bilinear" approximation to interactive dynamics

4. (Estimated in a Bayesian context, allowing formal comparison of any number/type of models…)
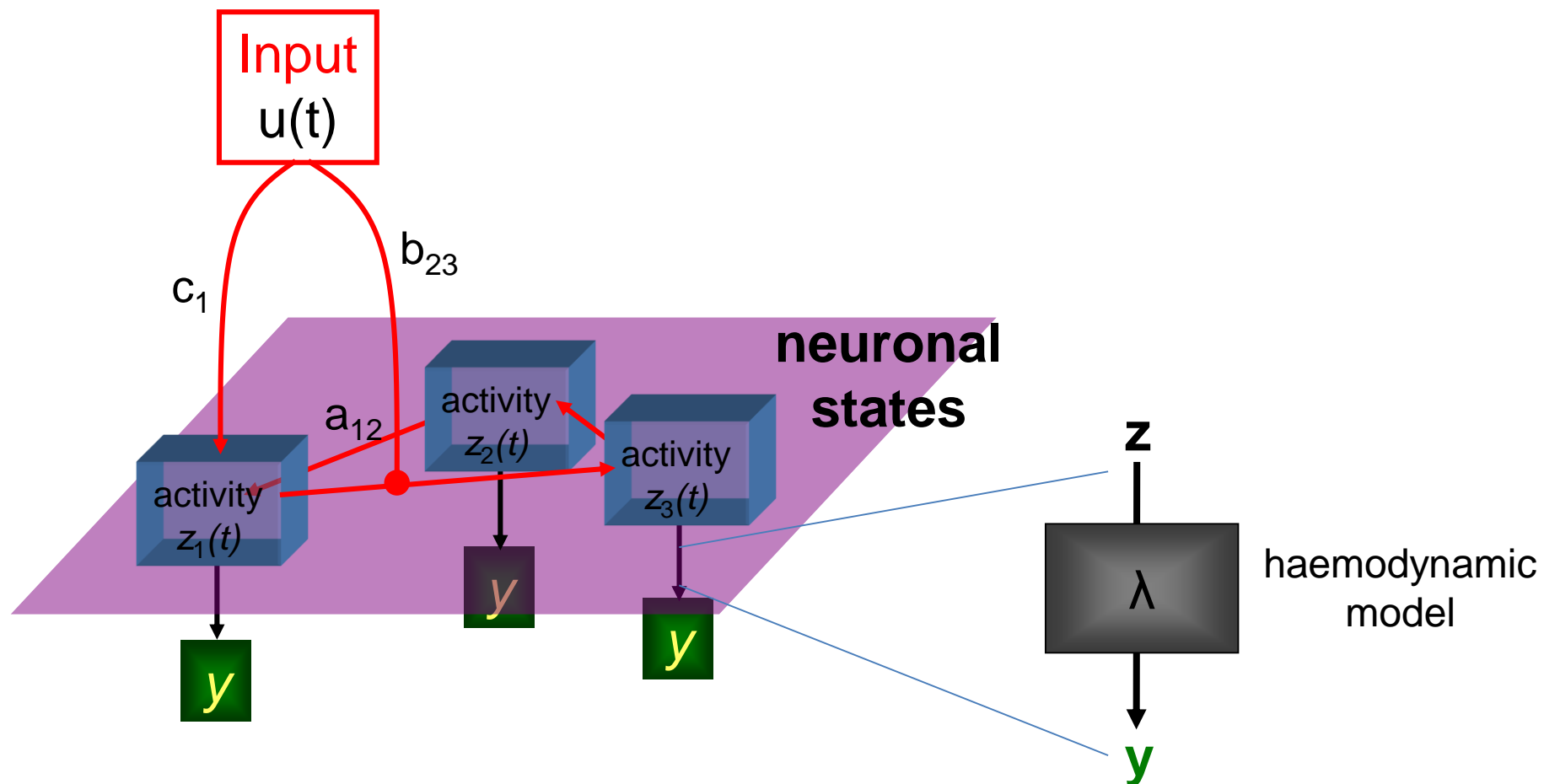
Rough comparison of popular methods?

	Experimental modulation	Temporal/ Dynamical	Network model	Haemodynamic Model (for fMRI)
Correlation / ICA / PCA				
PPI	Y			
Granger		Y		
SEM			Y	
DCM	Y	Y	Y	Y

DCM overview

Friston et al. 2003, *NeuroImage*

DCM Neural Level

Oridinary Differential Equations:

$$s$$

$$z_1 \quad \frac{dz_1}{dt} = -sz_1 \quad \Rightarrow \quad z_1(t) = z_1(0)\exp(-st), \quad z_1(0) = 1$$

Half-life τ

Decay function

$$z_1(\tau) = 0.5z_1(0)$$

$$= z_1(0)\exp(-s\tau)$$

$$\Rightarrow \quad s = \ln 2 / \tau$$

$$0.5z_1(0)$$

$$\tau = \ln 2 / s$$

DCM parameters = rate constants
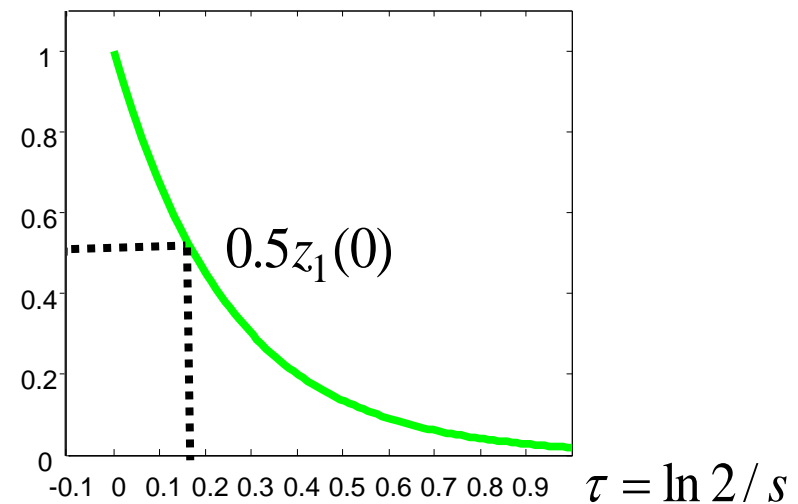
Neurodynamics:
2 nodes, 1 driving input
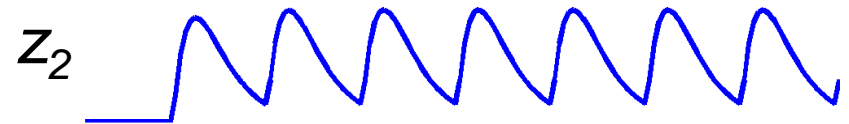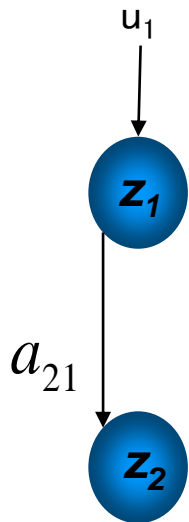
u_1

u_1

z_1
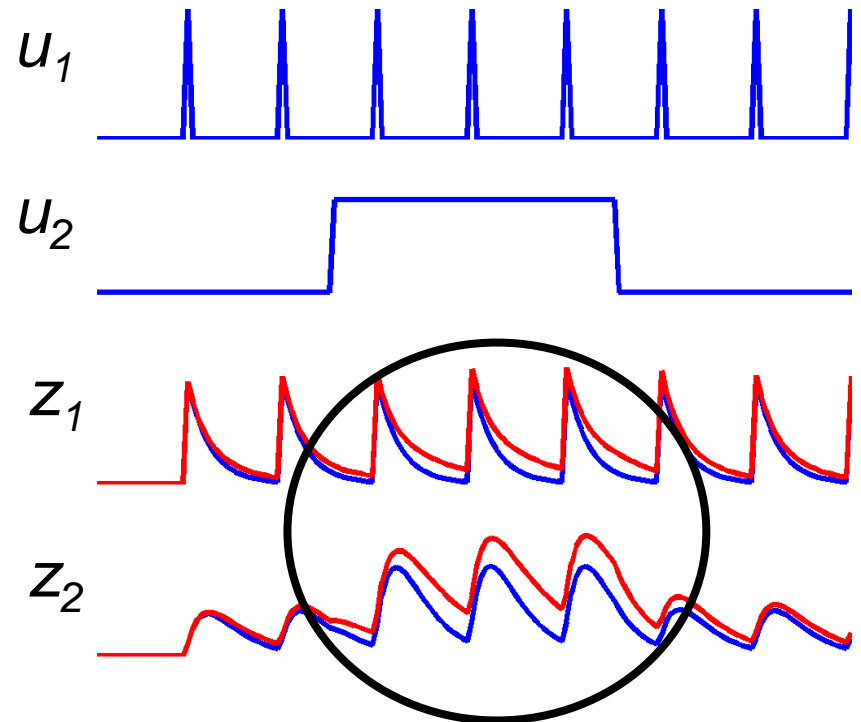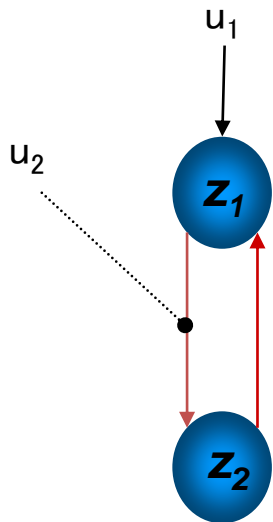
a_{21}

z_2

z_1

z_2

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = s \begin{bmatrix} -1 & 0 \\ a_{21} & -1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} c \\ 0 \end{bmatrix} u_1 \qquad a_{21} > 0$$

Neurodynamics:
...+1 modulatory input
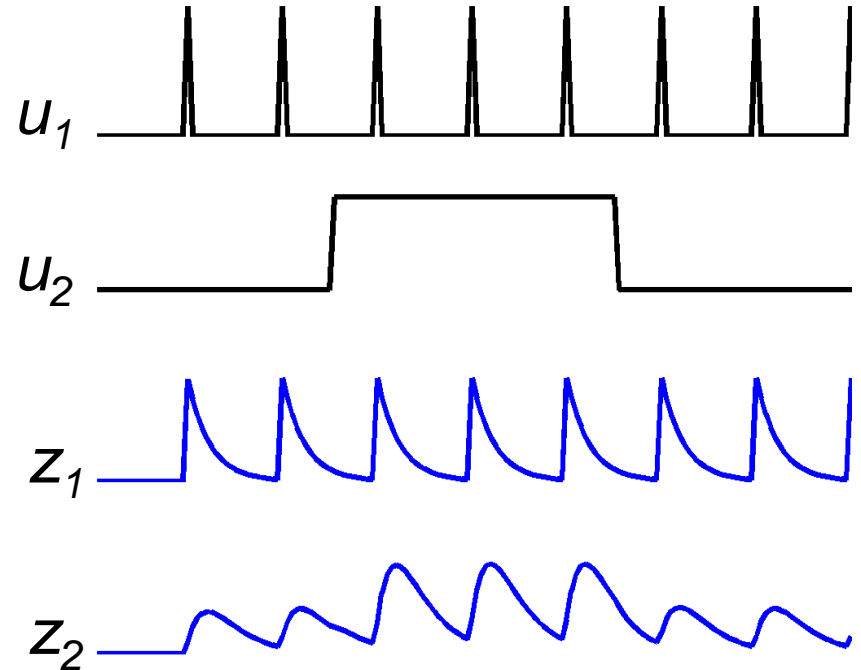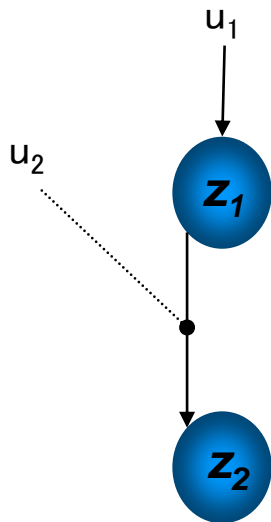
$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = s \begin{bmatrix} -1 & 0 \\ a_{21} & -1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + u_2 \begin{bmatrix} 0 & 0 \\ b_{21} & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} c \\ 0 \end{bmatrix} u_1 \qquad a_{21}, b_{21} > 0$$

Neurodynamics:
...+ reciprocal connections

u_1

u_2

z_1

z_2

reciprocal connection disclosed by u_2

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = s \begin{bmatrix} -1 & a_{12} \\ a_{21} & -1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + u_2 \begin{bmatrix} 0 & 0 \\ b_{21} & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} c \\ 0 \end{bmatrix} u_1 \qquad a_{12}, a_{21}, b_{21} > 0$$

Bilinear state equation

modulatory inputs

intrinsic connectivity

modulatory connectivity

direct inputs

driving inputs

$$\begin{bmatrix} \dot{z}_1 \\ \vdots \\ \dot{z}_n \end{bmatrix} = \left\{ \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} + \sum_{j=1}^{m} u_j \begin{bmatrix} b_{11}^{(j)} & \cdots & b_{1n}^{(j)} \\ \vdots & \ddots & \vdots \\ b_{n1}^{(j)} & \cdots & b_{nn}^{(j)} \end{bmatrix} \right\} \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix} + \begin{bmatrix} c_{11} & \cdots & c_{1d} \\ \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{nd} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_d \end{bmatrix}$$

n regions

m mod inputs

d drv inputs

$$\dot{z} = (A + \sum_{j=1}^{m} u_j B^{(j)})z + Cu$$
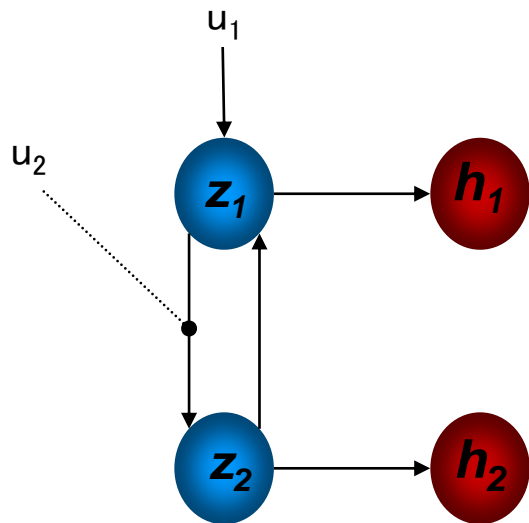
The haemodynamic "Balloon" model
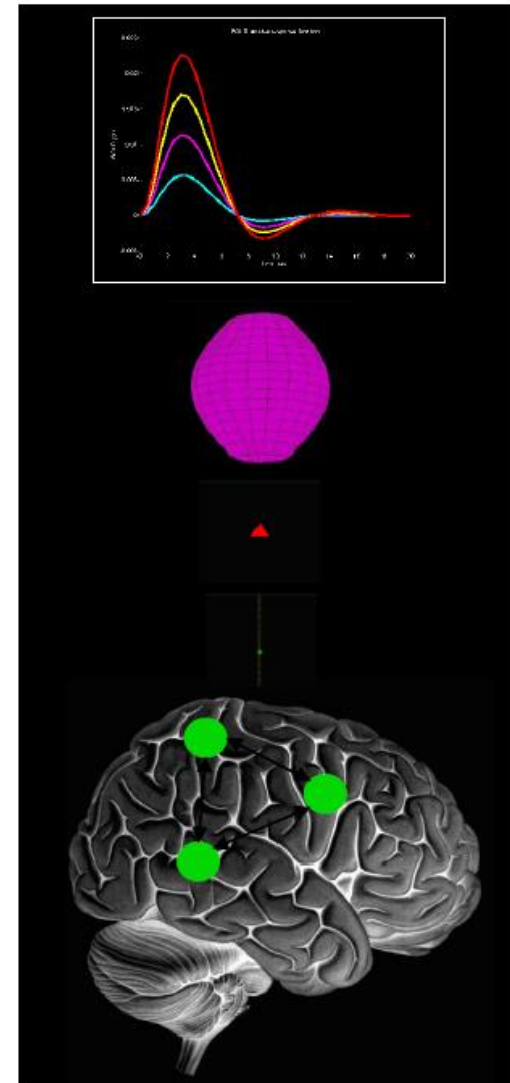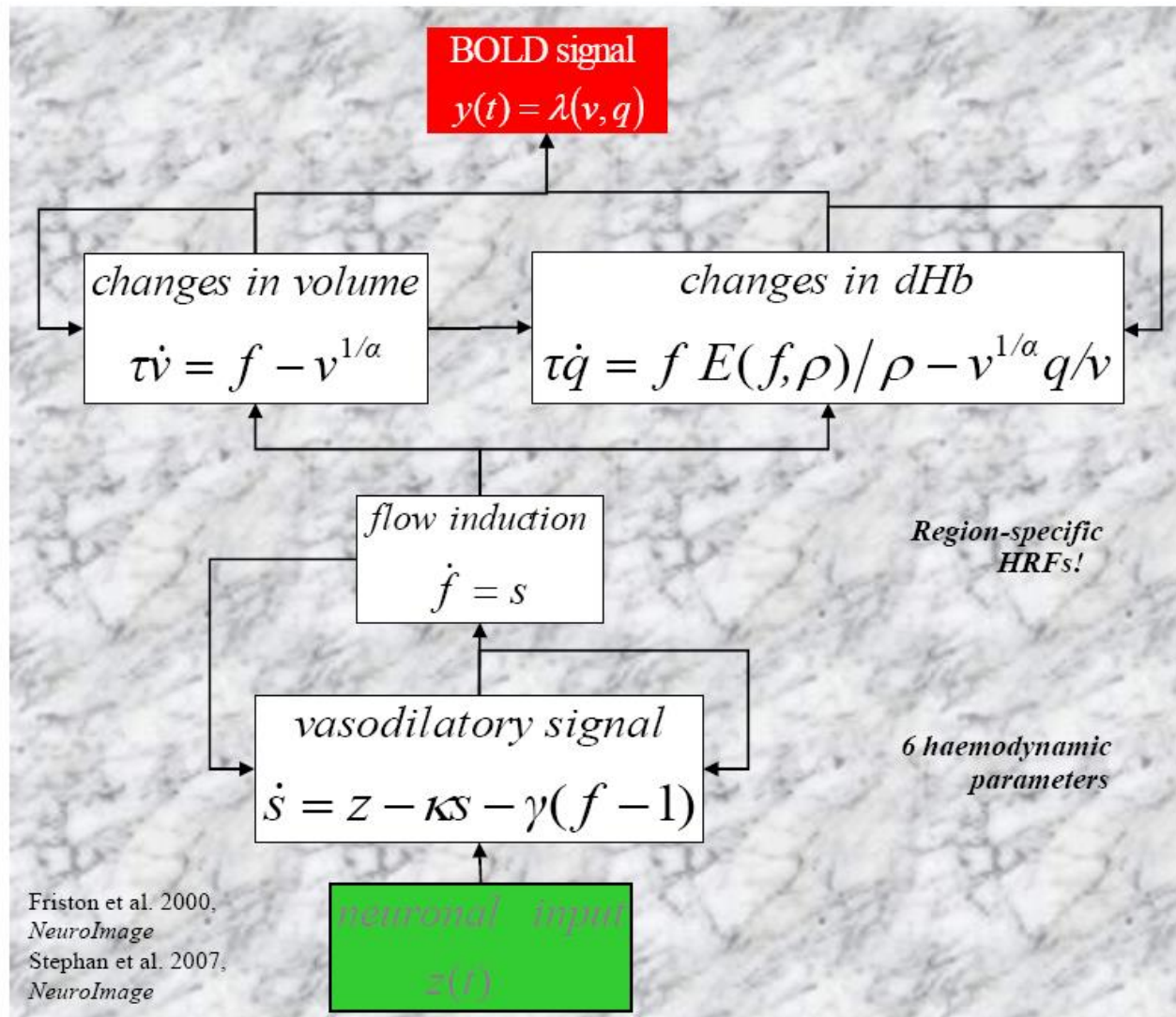
BOLD signal
$$y(t) = \lambda(v, q)$$

changes in volume
$$\tau \dot{v} = f - v^{1/\alpha}$$

changes in dHb
$$\tau \dot{q} = f E(f, \rho) / \rho - v^{1/\alpha} q / v$$

flow induction
$$\dot{f} = s$$

vasodilatory signal
$$\dot{s} = z - \kappa s - \gamma(f - 1)$$

neuronal impulse
$$z(t)$$

Region-specific HRFs!

6 haemodynamic parameters

Friston et al. 2000,
NeuroImage
Stephan et al. 2007,
NeuroImage

Haemodynamics: reciprocal connections

u_1

u_2

z_1 h_1

z_2 h_2

BOLD

(without noise)

BOLD

(without noise)

seconds
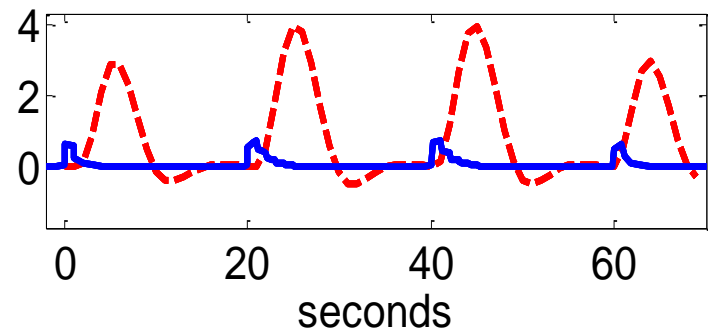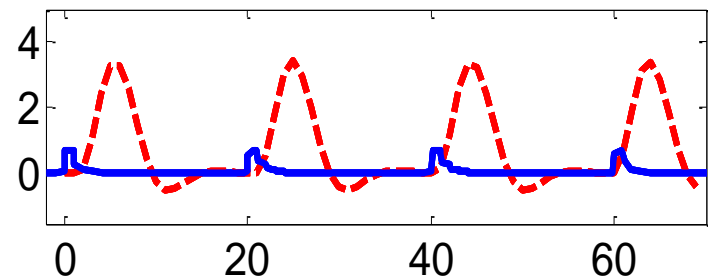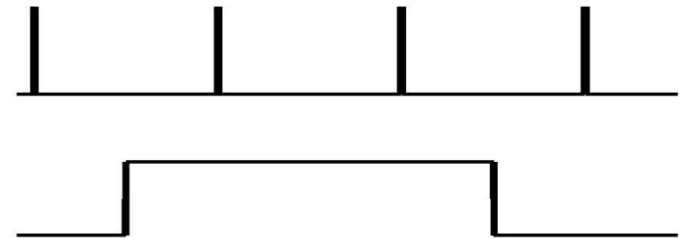
h(u,θ) represents the BOLD response (balloon model) to input

blue: neuronal activity

red: BOLD response
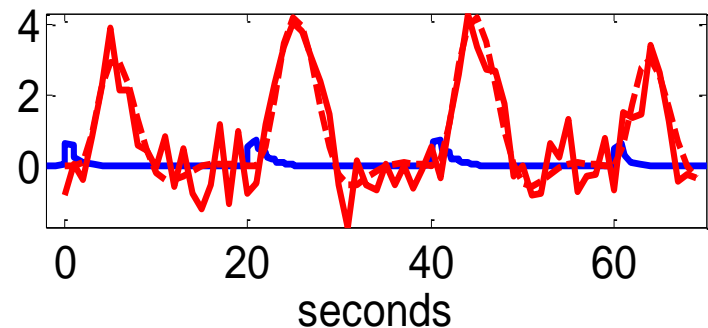
Haemodynamics: reciprocal connections
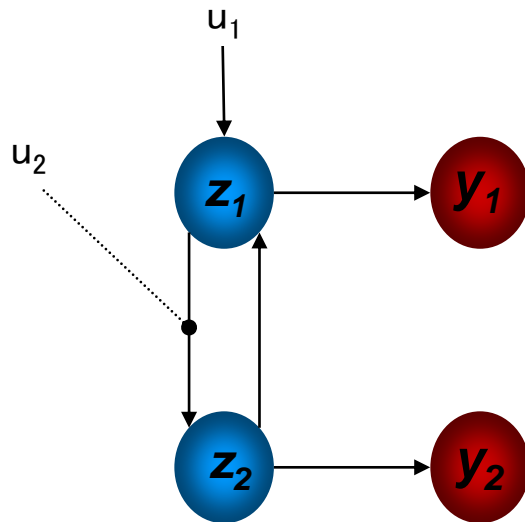
BOLD

with

Noise added

BOLD

with

Noise added

seconds
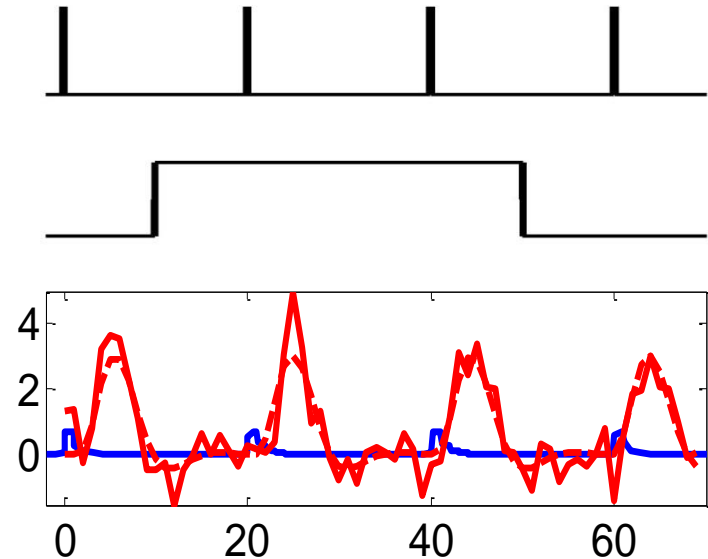
y represents simulated observation of BOLD response, i.e. includes noise

$$y = h(u, \theta) + e$$

Conceptual overview

Neuronal state equation $\dot{z} = F(z, u, \theta^n)$

The bilinear model $\dot{z} = (A + \sum u_j B^j)z + Cu$

effective connectivity

$$A = \frac{\partial F}{\partial z} = \frac{\partial \dot{z}}{\partial z}$$

modulation of connectivity

$$B^j = \frac{\partial^2 F}{\partial z \partial u_j} = \frac{\partial}{\partial u_j} \frac{\partial \dot{z}}{\partial z}$$

direct inputs

$$C = \frac{\partial F}{\partial u} = \frac{\partial \dot{z}}{\partial u}$$

Input
u(t)

c_1

b_{23}

a_{12}

neuronal states

activity $z_1(t)$

activity $z_2(t)$

activity $z_3(t)$

y

y

y

BOLD

z

λ

haemodynamic model

y
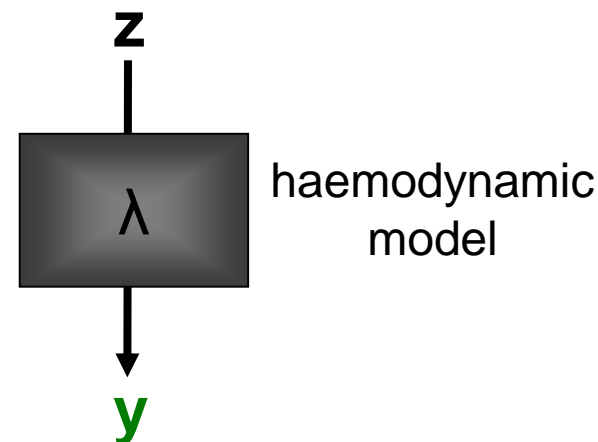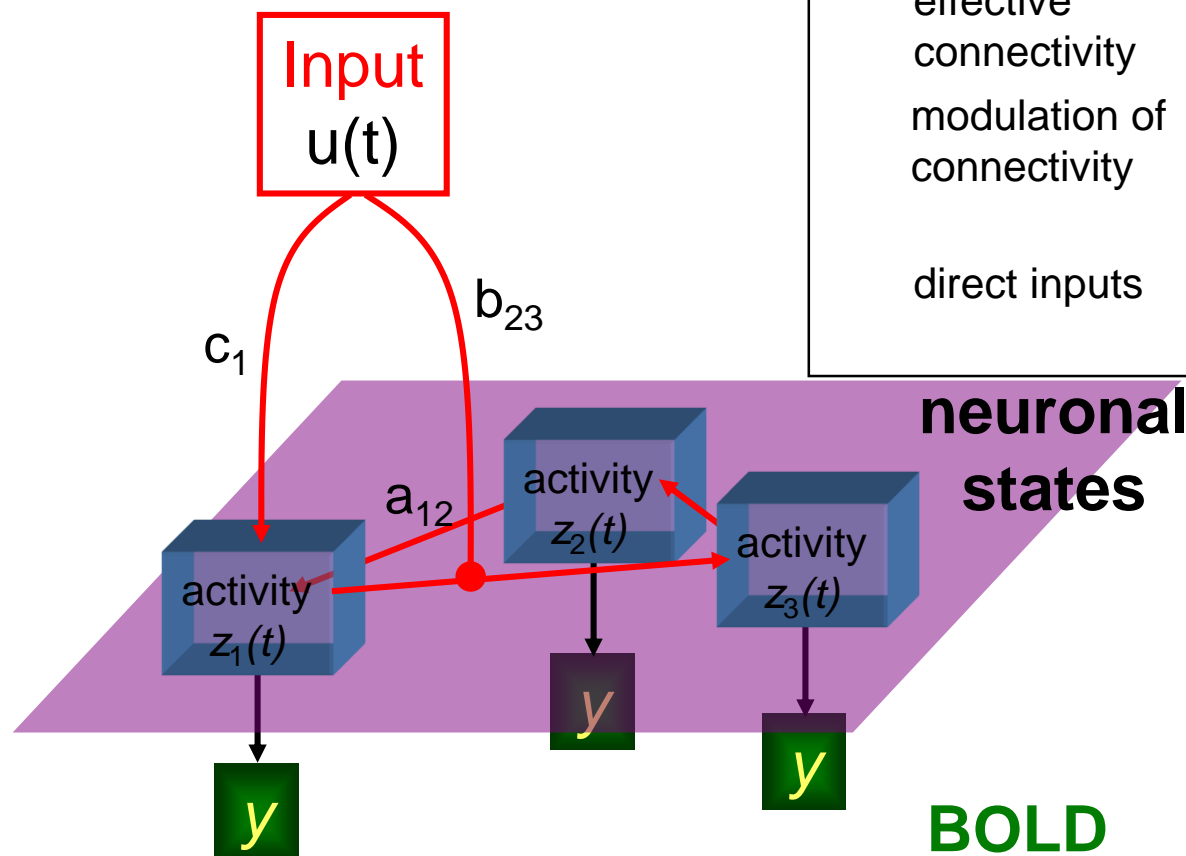
Friston et al. 2003, *NeuroImage*

Inference on model space
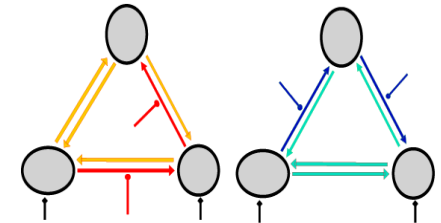
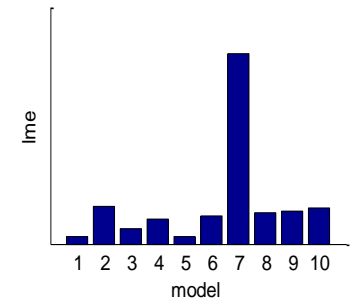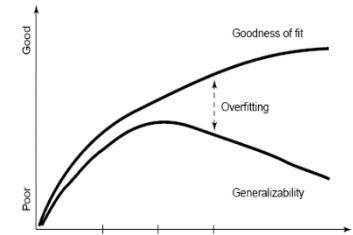Model evidence: The optimal balance of fit and complexity

Comparing models

- Which is the best model?

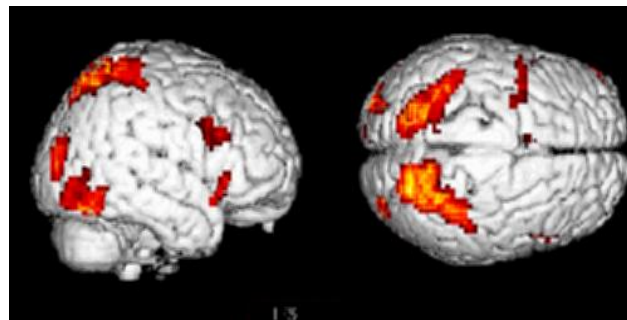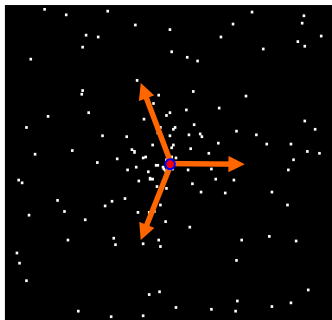Comparing families of models

- What type of model is best?

    - Feedforward vs feedback

    - Parallel vs sequential processing

    - With or without modulation
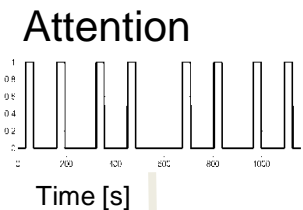
Only compare models with the same data
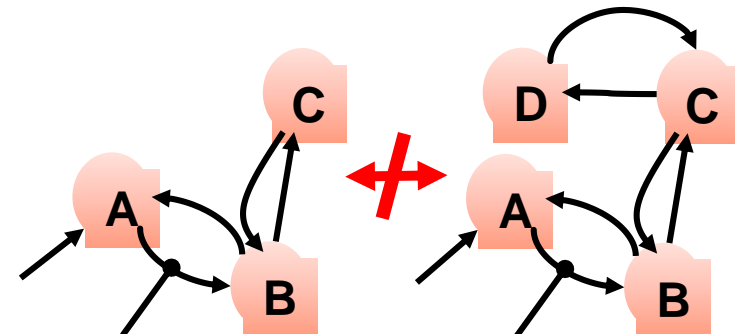
Example DCM:
Attention to motion

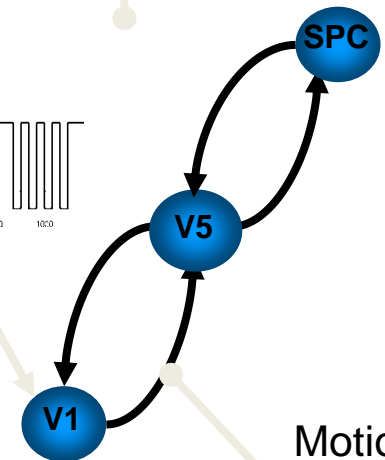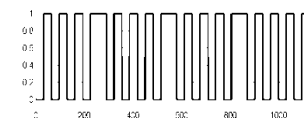What is site of **attention modulation** during *visual motion processing*

Attention

Time [s]

?

SPC

Photic

V5

- fixation only
- observe static dots   + photic   → V1
- observe moving dots   + motion   → V5
- task on moving dots   + attention   → V5 + parietal cortex

V1

Motion

Friston et al. 2003, *NeuroImage*
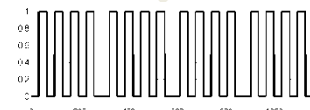
Example DCM:
Attention to motion

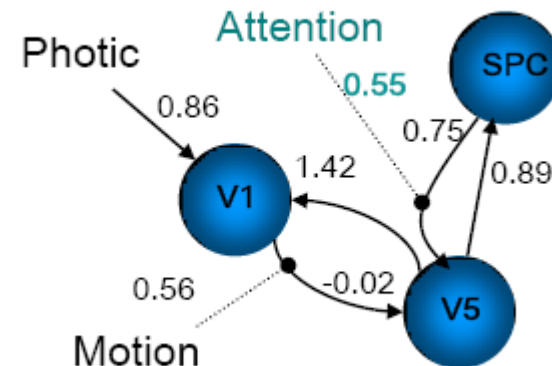**Model 1:**
attentional modulation
of V1→V5
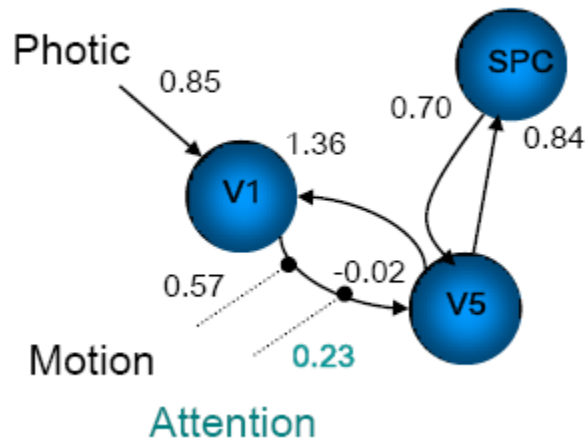
Model 2:
attentional modulation
of SPC→V5

Bayesian model selection: Model 1 better than model 2

$$\log p(y \mid m_1) \gg \log p(y \mid m_2)$$

→ attention primarily modulates V1→V5 (in these data)

So, DCM···.

- enables one to **infer hidden neuronal processes**

- allows one to **test mechanistic hypotheses** about observed effects

  - uses a deterministic differential equation to model neuro-dynamics (represented by matrices A, B and C)

- is informed by anatomical and physiological principles

- uses a **Bayesian framework** to estimate model parameters

- is a generic approach to modelling experimentally perturbed dynamic systems

  - provides an observation model for neuroimaging data, e.g. fMRI, M/EEG

  - DCM is **not model or modality specific** (models will change and the method extended to other modalities e.g. LFPs)

Variants of DCM

- DCM for fMRI

  – "non-linear" DCM: modulatory input (B) equal to activity in another region

  – "two-state" DCM: inhibitory and excitatory neuronal subpopulations

  – "stochastic" DCM: random element to activity (e.g, for resting state)

- DCM for E/MEG

  – "evoked" responses (complex neuronal model based on physiology)

  – "induced" responses (within/across frequency power coupling; no physiological model (more like DCM for fMRI))

  – "steady-state" responses

  – with (e.g, EEG/MEG) or without (e.g, LFP, iEEG) a forward (head) model

Functional/Effective Connectivity for MEG/EEG

Rik Henson

MRC CBU, Cambridge

Functional Connectivity Background

- Much interest in functional connectivity in fMRI

- And yet many neural interactions (e.g, coupled oscillations) occur at a timescale faster than visible by fMRI

- So, real promise of MEG/EEG is functional connectivity?

Talk Overview

1. Problem of Field Spread (Volume Conduction)

2. Linear vs Nonlinear measures

3. Directed vs Undirected measures

4. Direct vs Indirect measures

5. Generative Models

Field Spread Problem

Many (zero-lag) measures of functional connectivity between sensors can be spurious, i.e, reflect activity from single source

No true source connectivity

True source connectivity
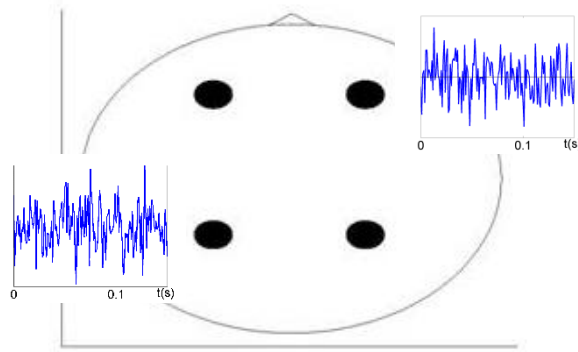
Field Spread Problem
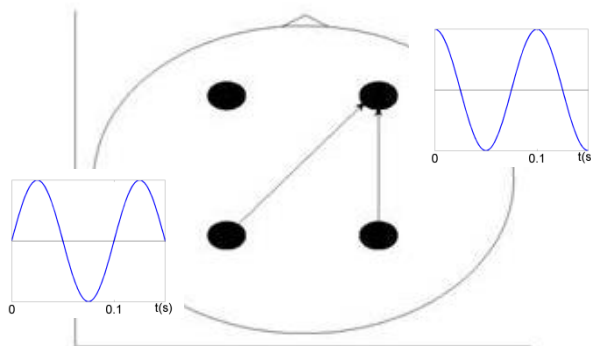
Source reconstruction reduces field spread problem…

…and allows easier comparison with fMRI connectivity

BUT spurious connections between sources can remain ("point-spread")

Hillebrand et al (2012) Neuroimage

One approach is to orthogonalise raw data, then correlate (0-lag) power envelopes…

Colclough et al (2015) Neuroimage

…another uses fact that field-spread is instantaneous, so time- or phase-lagged measures are immune to field spread (though assume no true zero-lag connectivity)

Different Types of Connection

Undirected, Indirect (bivariate)
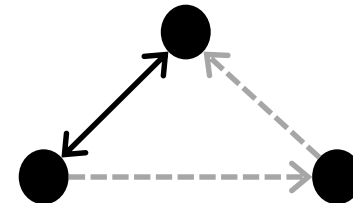
Directed, Indirect (bivariate)
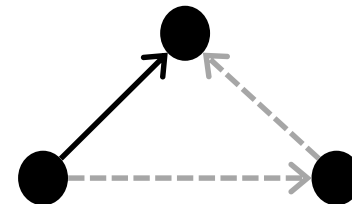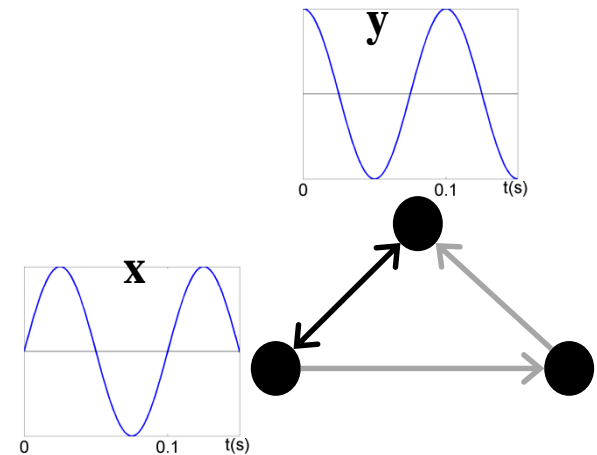
Directed, Direct (multivariate) ("effective connectivity")
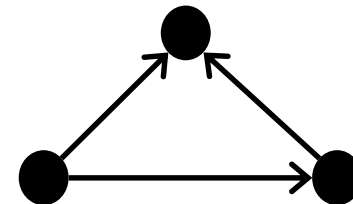
Cross-Correlation

Undirected, Indirect, Linear (sensitive to Field-spread when $l=0$)

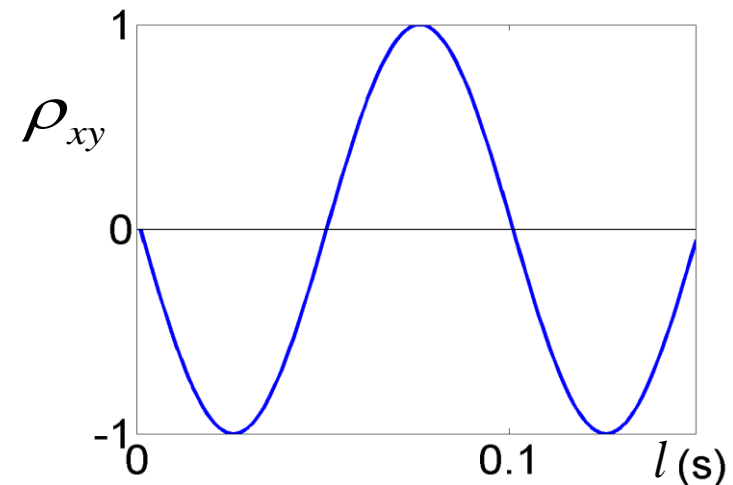$$c_{xy}(l) = \left\langle \left(x_t - \bar{x} \right) \left(y_{t+l} - \bar{y} \right) \right\rangle_t$$

Cross-covariance

$l = $ "lag"

$$\rho_{xy}(l) = \frac{c_{xy}(l)}{\sigma_x \sigma_y}$$

Cross-correlation

Coherency
(Fourier transform of cross-covariance)

Undirected, Indirect, Linear, sensitive to Field-spread

$$c_{xy}(l) = \left\langle \left(x_t - \bar{x}\right)\left(y_{t+l} - \bar{y}\right) \right\rangle_t$$

Cross-covariance

$$C_{xy}(f) = \sum_l c_{xy}(l) e^{-2\pi i.l.f}$$

Coherency

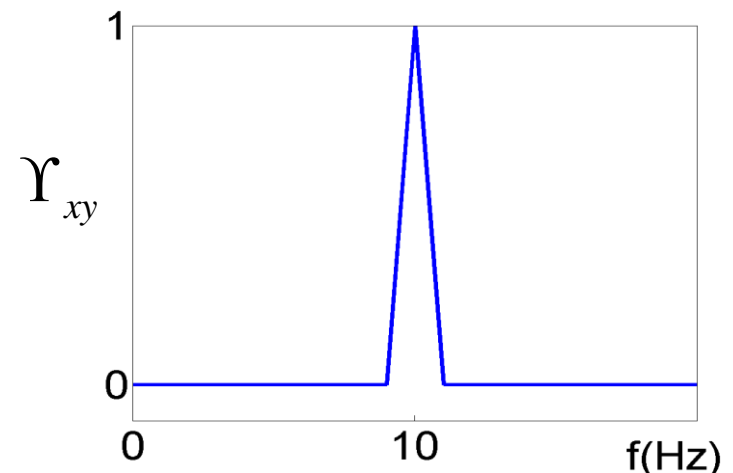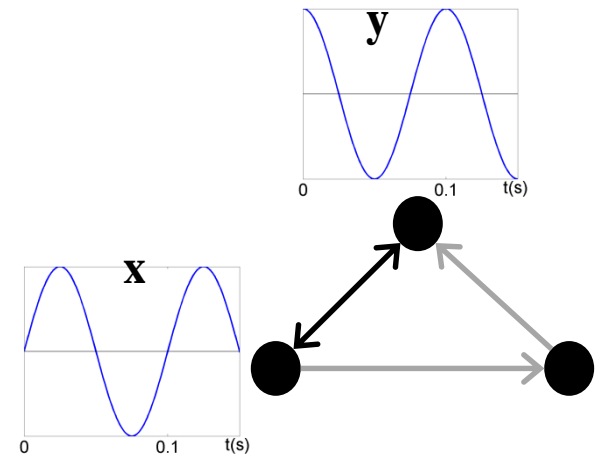$$\Upsilon_{xy}(f) = \frac{\left|C_{xy}(f)\right|^2}{\left|C_{xx}(f)\right|\left|C_{yy}(f)\right|}$$

(Magnitude-squared) Coherence
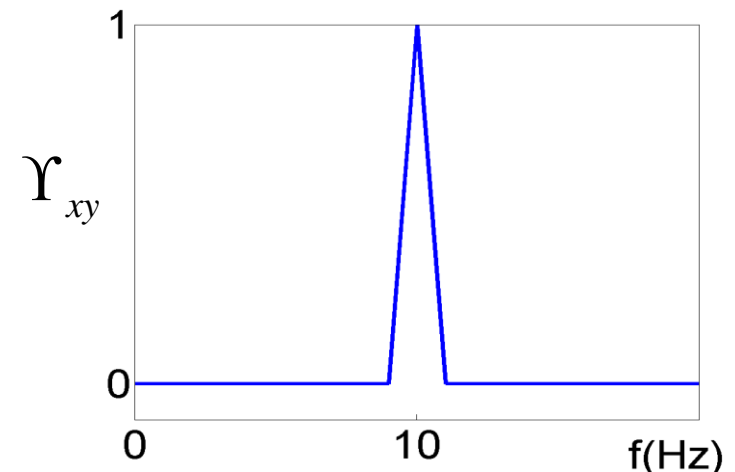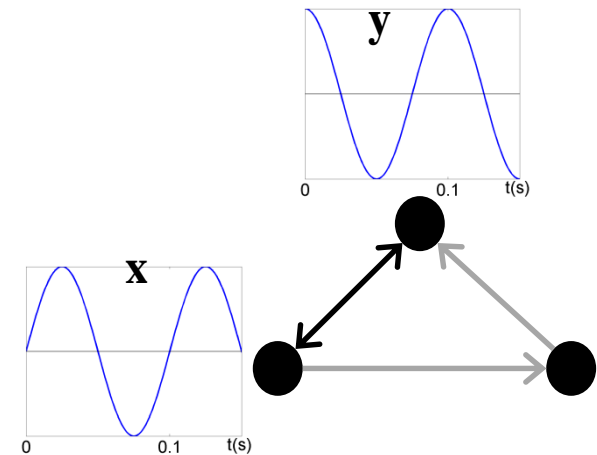
Digression on Complex Numbers

An oscillation of frequency *f* can be represented in terms of amplitude and phase (polar coordinates), which can also be represented by a complex number
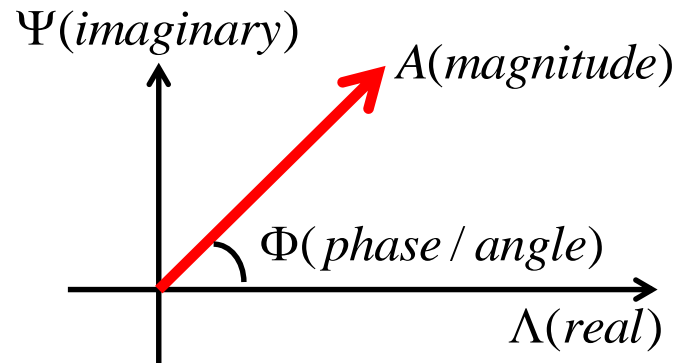
$$C(f) = A(f)e^{i\Phi(f)}$$

$$= \Lambda(f) + i\Psi(f)$$

$$A(f) = |C(f)| = \sqrt{\Lambda^2(f) + \Psi^2(f)} \qquad \Phi(f) = \arctan(\Psi(f)/\Lambda(f))$$

Coherence

Undirected, Indirect, Linear, sensitive to Field-spread

$$c_{xy}(l) = \left\langle \left(x_t - \bar{x} \right) \left(y_{t+l} - \bar{y} \right) \right\rangle_t$$

Cross-covariance

$$C_{xy}(f) = \sum_l c_{xy}(l) e^{-2\pi i . l . f}$$

Coherency

$$\Upsilon_{xy}(f) = \frac{\left| C_{xy}(f) \right|^2}{\left| C_{xx}(f) \right| \left| C_{yy}(f) \right|}$$

(Magnitude-squared) Coherence

Imaginary Coherency

Undirected, Indirect, Linear, immune to Field-spread

$$c_{xy}(l) = \left\langle \left(x_t - \bar{x}\right)\left(y_{t+l} - \bar{y}\right)\right\rangle_t$$
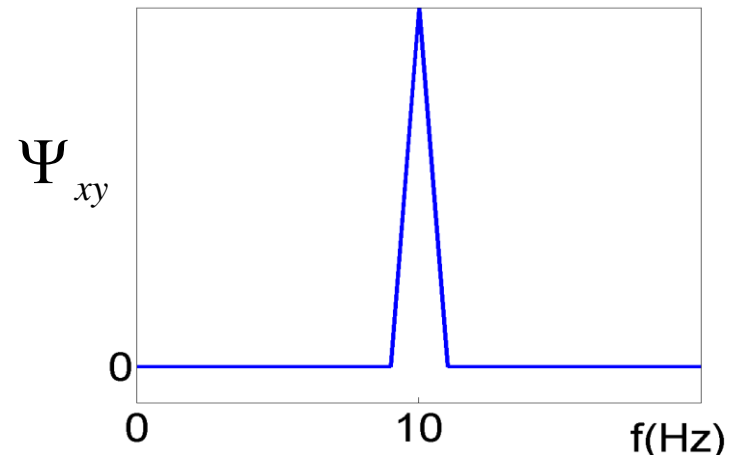
$$C_{xy}(f) = \sum_l c_{xy}(l)e^{-2\pi i.l.f}$$
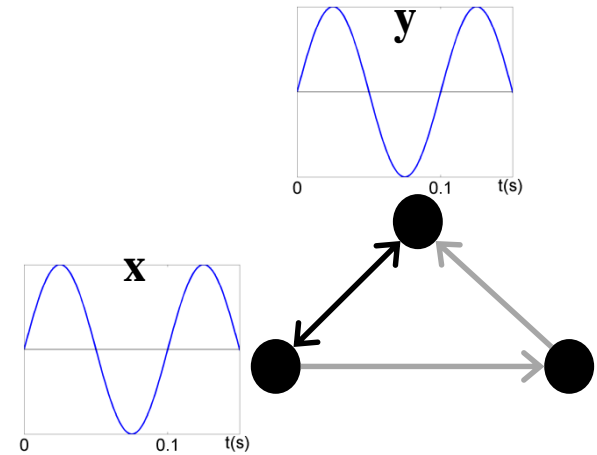
Coherency
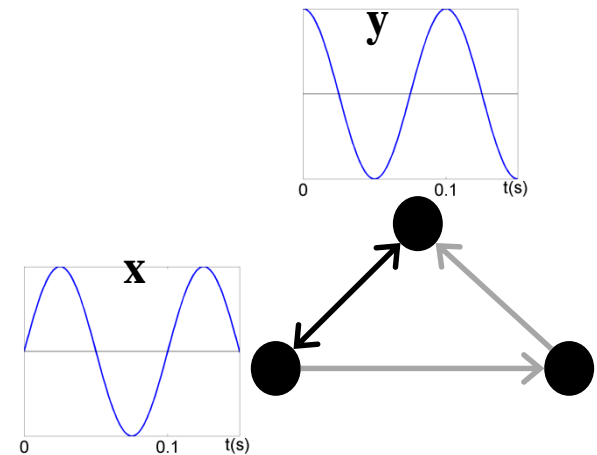
$$\Psi_{xy}(f) = imag(C_{xy}(f))$$

Imaginary Coherency

Nolte et al (2004) Clin Neurophys

Imaginary Coherency

A zero imaginary component implies a phase of the coherency of either 0° or 180°, which could be caused by field-spread…

$$\Psi = 0$$

$$\Lambda > 0$$

$$\Psi_{xy}(f) = imag(C_{xy}(f))$$

Nolte et al (2004) Clin Neurophys

Imaginary Coherency

MRC | Cognition and Brain Sciences Unit

A zero imaginary component implies a phase of the coherency of either 0° or 180°, which could be caused by field-spread…

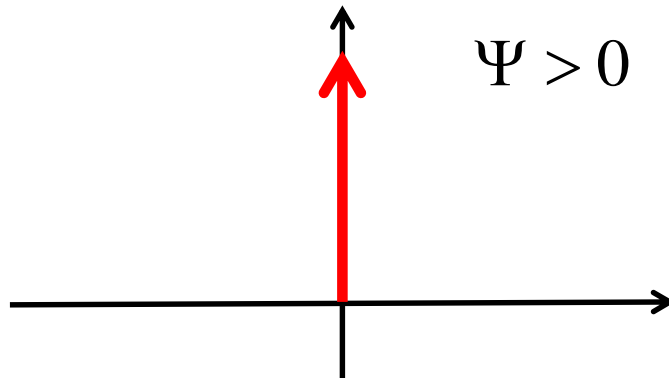$$\Psi = 0$$

$$\Lambda < 0$$

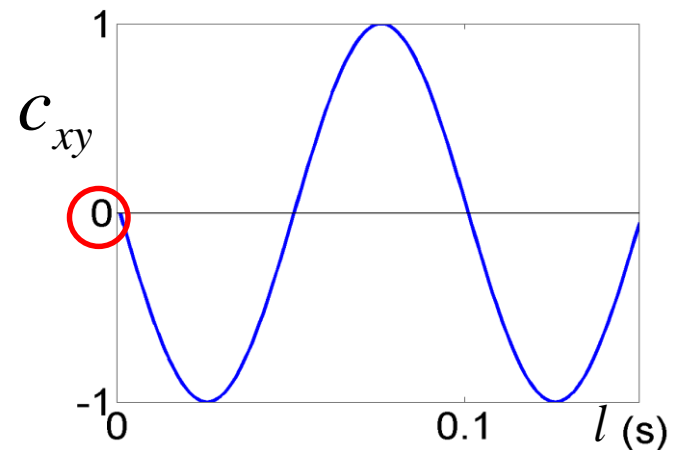$$\Psi_{xy}(f) = imag(C_{xy}(f))$$

Nolte et al (2004) Clin Neurophys
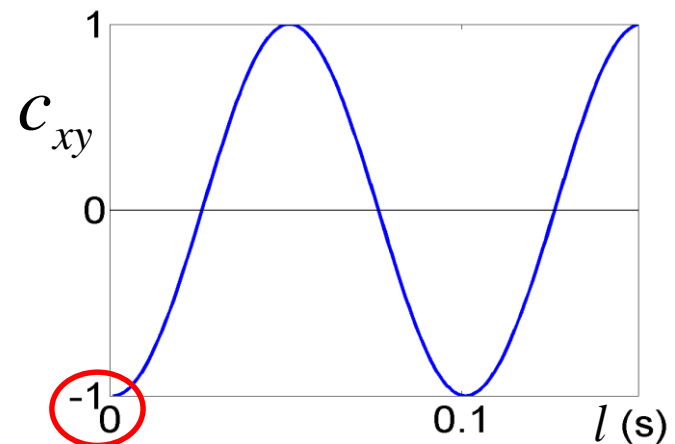
Imaginary Coherency

…whereas a NON-zero imaginary component implies a phase of the coherency other than 0° or 180°, which can NOT be caused by field-spread

$$\Psi > 0$$

$$\Psi_{xy}(f) = imag(C_{xy}(f))$$
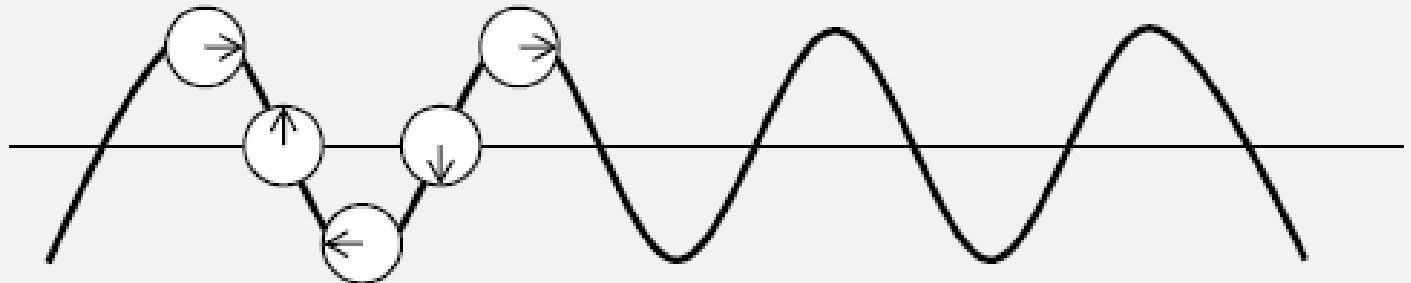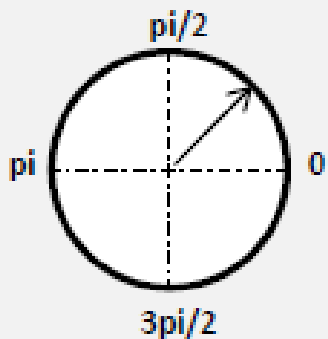
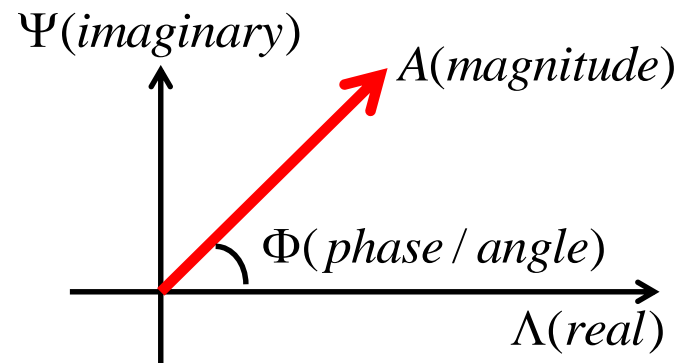*Nolte et al (2004) Clin Neurophys*
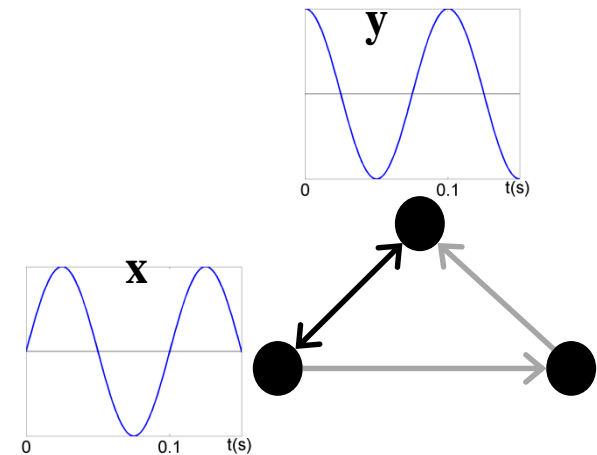
Digression on Analytic Signals

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A signal can be represented analytically in terms of its amplitude and phase over time (within a narrow frequency band) (e.g, using Hilbert transform)
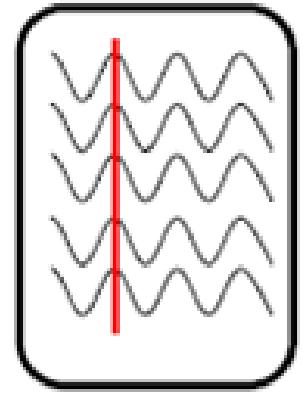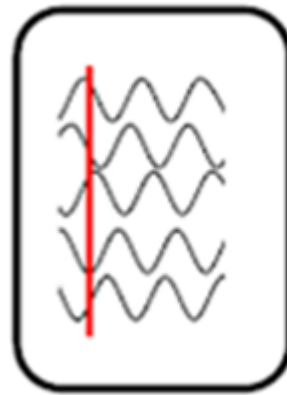
$$x(t, f) = A(t, f)e^{i\Phi(t,f)}$$

$\Psi(imaginary)$

$A(magnitude)$

$\Phi(phase / angle)$

$\Lambda(real)$

Phase-related Measures

Undirected, Indirect, Linear, immune to Field-spread (when $\Delta\Phi \neq 0$)

$$x(t) = A_x(t)e^{i\Phi_x(t)}$$
$$y(t) = A_y(t)e^{i\Phi_y(t)}$$

$$\Delta\Phi(t) = \Phi_x(t) - \Phi_y(t)$$

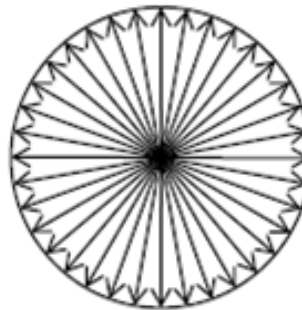$$PLV = \left\langle e^{i\Delta\Phi(t)} \right\rangle_t$$
Phase-Locking Value

$$PLI = \left\langle sign(\Delta\Phi(t)) \right\rangle_t$$
Phase-Lag Index

PLV=0   PLV=0.5   PLV=0.75

Stam et al (2007) Human Brain Mapp
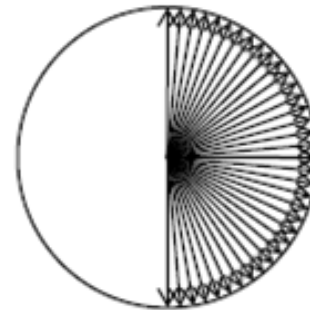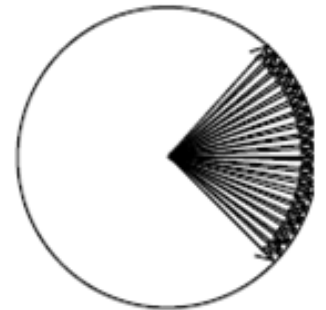
Cross-frequency coupling

$x(t)$
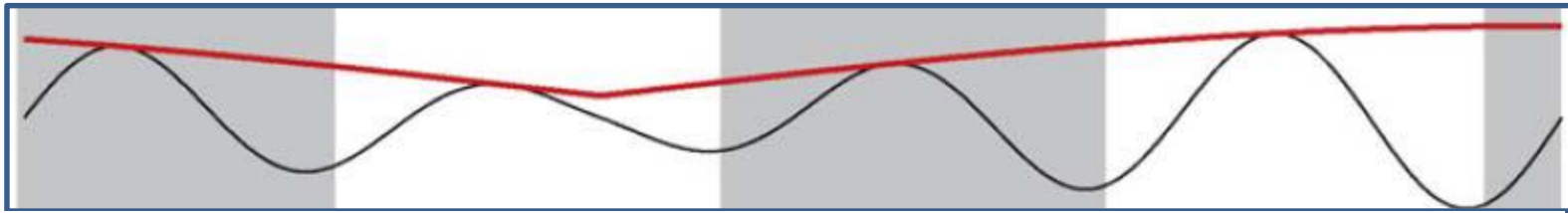
$y(t)$

Power-Power
$A_x(t) : A_y(t)$

Phase-Phase
$\Phi_x(t) : \Phi_y(t)$

Phase-Freq
$\Phi_x(t) : F_y(t)$

Phase-Power
$\Phi_x(t) : A_y(t)$

Talk Overview

1. Problem of Field Spread (Volume Conduction)

2. Linear vs Nonlinear measures

3. Directed vs Undirected measures

4. Direct vs Indirect measures

5. Generative Models

Nonlinear Measures

Nonlinear Measures
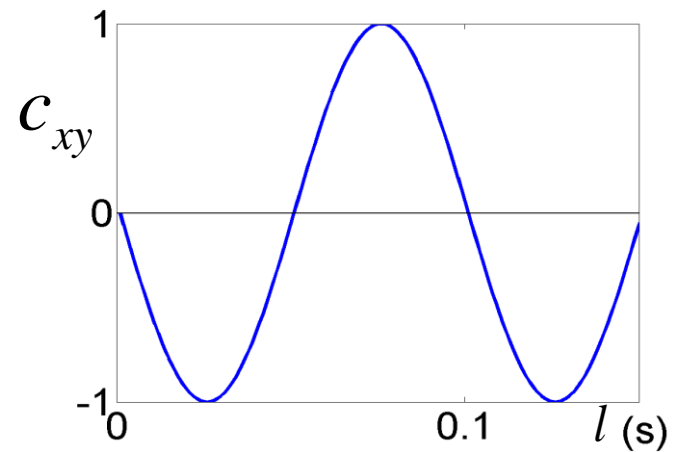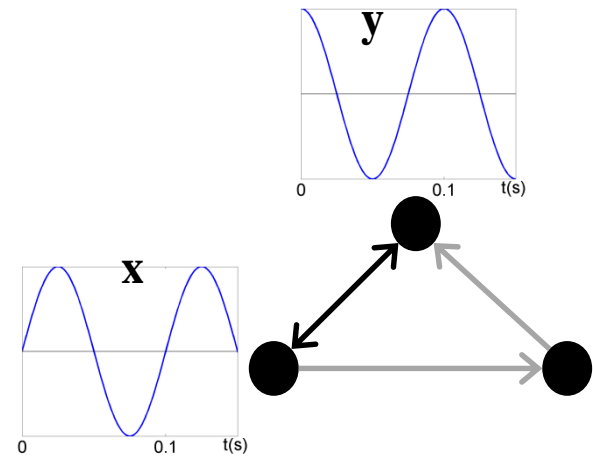
Cross-correlation/coherence insensitive to nonlinear dependencies

$$\mathbf{y} = x^2$$

$$\mathbf{x}$$

$$c_{xy}$$

Mutual Information

$$MI(x,y) = \sum_{x,y} p(x,y) \log\left(\frac{p(x,y)}{p(x)p(y)}\right)$$

$\mathbf{y} = x^2$

$\mathbf{x}$

$p(x,y)$

$p(x)$

Mutual Information

Sensitive to Field-spread, Undirected, Indirect, Nonlinear

$$MI(x, y) = \sum_{x,y} p(x, y) \log \left(\frac{p(x, y)}{p(x) p(y)} \right)$$
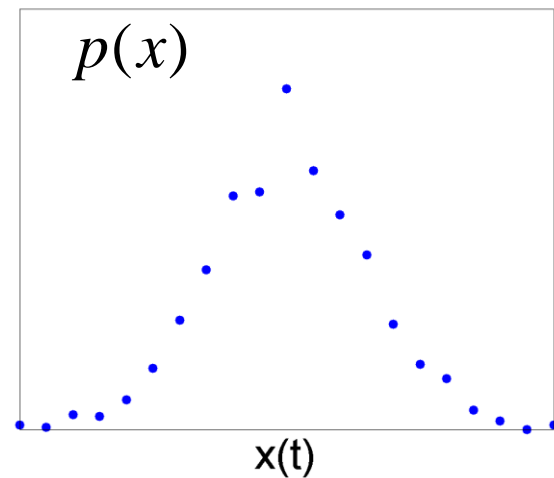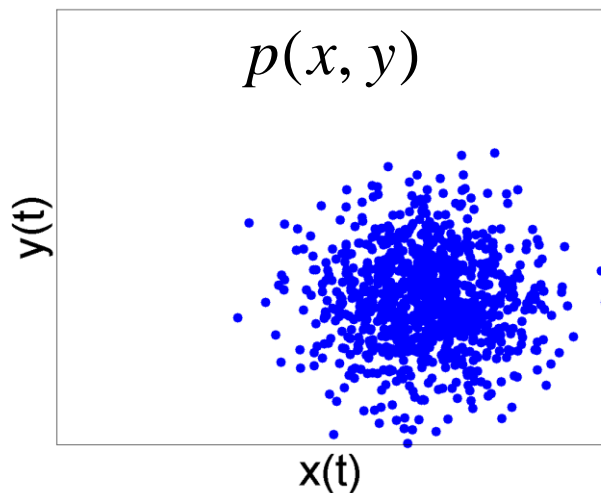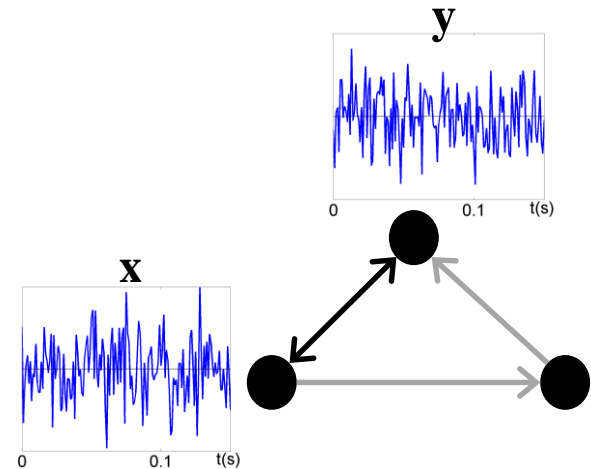
$p(x, y)$

$p(x)$

Talk Overview
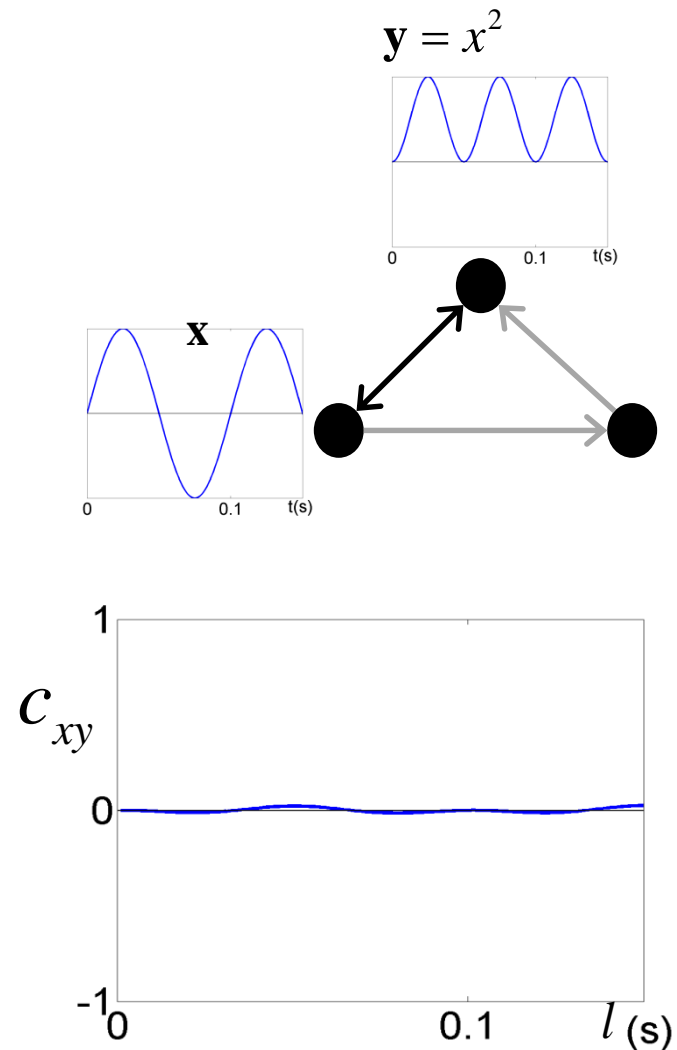
1. Problem of Field Spread (Volume Conduction)

2. Linear vs Nonlinear measures

3. Directed vs Undirected measures

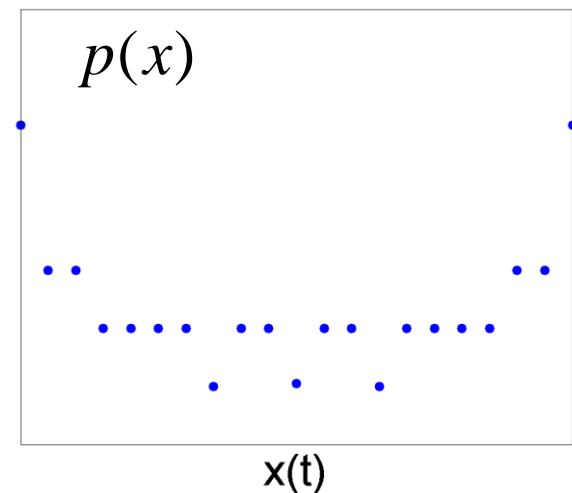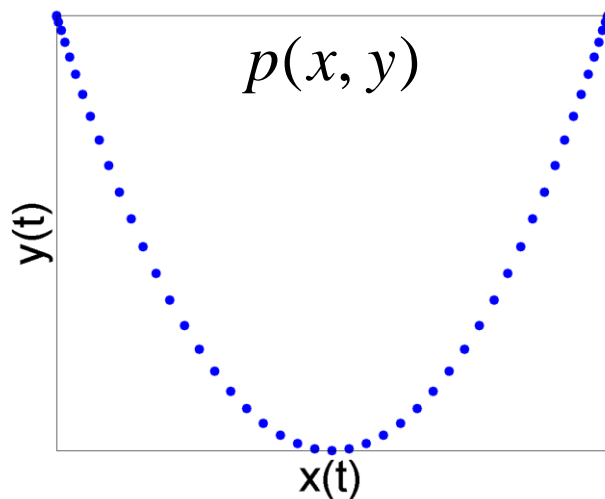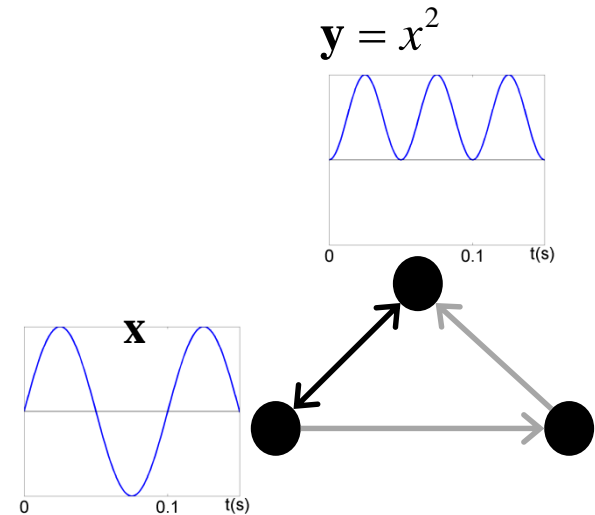4. Direct vs Indirect measures

5. Generative Models

Directed Measures

(bivariate) Granger Causality
Immune to Field-spread, Directed, Indirect, Linear

Auto-regressive model to order p
(assuming mean-corrected, with residuals e)

$$y_y(t) = a_1 y(t-1) + \ldots + a_p y(t-p) + e(t)$$

$$= \sum_{l=1}^{p} a_l y(t-l) + e(t)$$
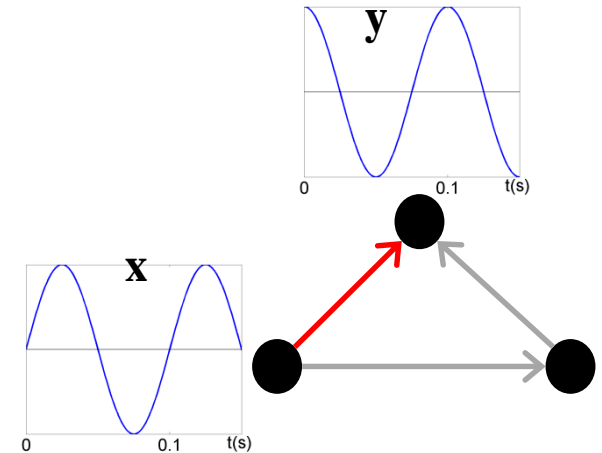
Augmented model including past values of x (to order q)

$$y_{y \leftarrow x}(t) = \sum_{l=1}^{p} a_l y(t-l) + \sum_{l=1}^{q} b_l x(t-l) + e(t)$$

If classical F-test shows b parameters are non-zero, then x "Granger-causes" y
(special case of MVAR; see later)

Directed, Nonlinear Measures

Transfer Entropy (lagged generalisation of mutual information)

Immune to Field-spread, Directed, Indirect, Nonlinear

$$TE_{y \to x}(l) = \sum_{x_{n+l}, x_n, y_n} p(x_{n+l}, x_n, y_n) \log \left(\frac{p(x_{n+l} \mid x_n, y_n)}{p(x_{n+l} \mid x_n)} \right)$$

$$TE_{x \to y}(l) = \sum_{y_{n+l}, y_n, x_n} p(y_{n+l}, x_n, y_n) \log \left(\frac{p(y_{n+l} \mid x_n, y_n)}{p(y_{n+l} \mid y_n)} \right)$$

Schreiber (2000) Phys Rev Letters

Generalised Synchronisation

Sensitive to Field-spread, Directed, Indirect, Nonlinear

$$x_t = [x_t, x_{t+l}, ..., x_{t+(m-1)l}]$$

$$y_t = [y_t, y_{t+l}, ..., y_{t+(m-1)l}]$$

m is the embedding dimension and l lag

$$S(x \mid y) = \frac{1}{N} \sum_{t=1}^{N} \frac{D_t(x)}{D_t(x \mid y)}$$

D is the Euclidean distance between x_t and embedded neighbours

Quian Quiroga et al (2000) Phys Rev E

Talk Overview

1. Problem of Field Spread (Volume Conduction)

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Direct Measures

Multivariate Autoregressive Modelling (MVAR)
Immune to Field-spread, Directed, Direct, Linear

$$X_i(t) = \sum_{j=1}^{N} \sum_{l=1}^{p} a_{ij}(l) X_j(t-l) + u_i(t)$$

$X_2(t)$

$X_1(t)$

$X_3(t)$

$a_{31}(l)$

Various summary measures, eg,
Partial Directed Coherence (PDC):

$$PDC_{ij}(f) = \frac{A_{ij}(f)}{\sqrt{\sum_{k=1}^{M} \left| A_{kj}(f) \right|^2}}$$

$$A_{ij}(f) = F(a_{ij}(l))$$

Generalised form of Granger Causality

Though insensitive to true zero-lag dependencies (occur in reality?)

Baccala & Sameshima (2001) Biol Cybernet

Talk Overview
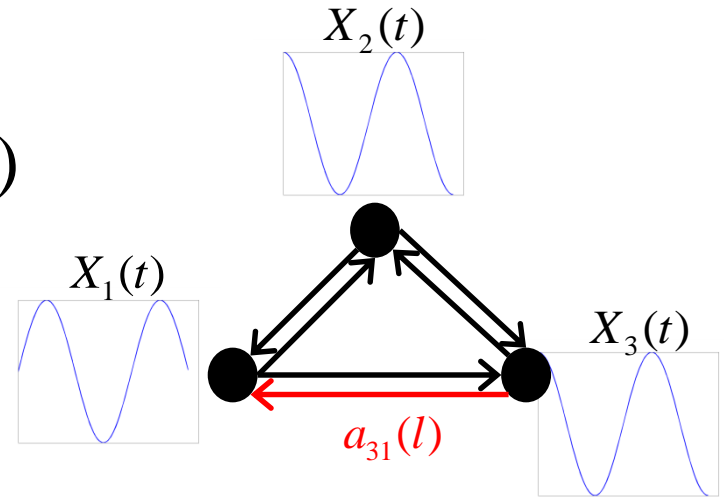
1. Problem of Field Spread (Volume Conduction)

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Generative Models

Immune to Field-spread, Directed, Direct, Nonlinear, model-driven
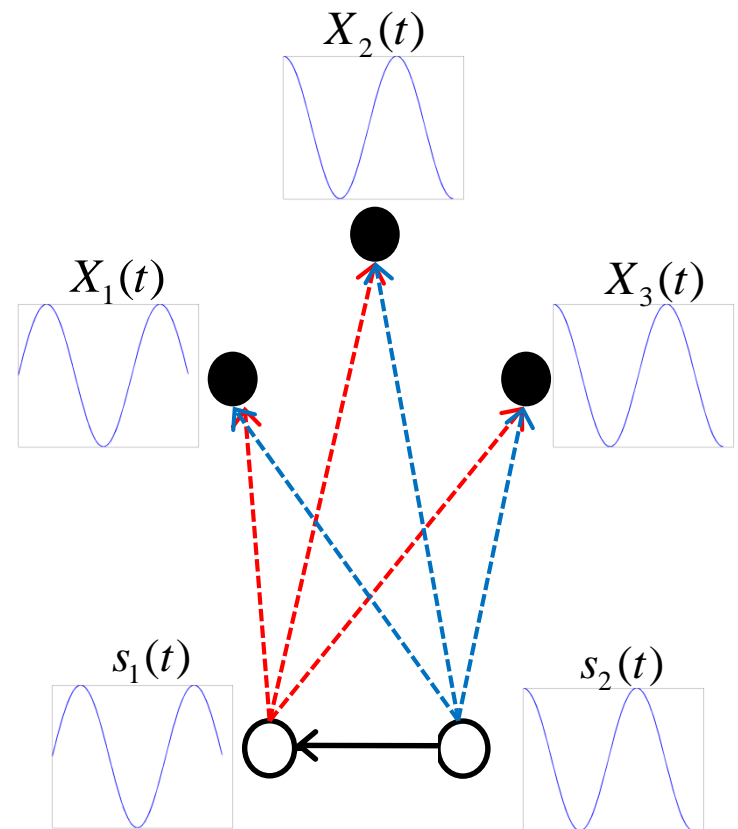
Connectivity modelled between sources

Projected to sensors via headmodel

Typically a handful of sources, and a range of networks fit to data

Bayesian methods for comparing which network model is best

Dynamic Causal Modelling (DCM) is one approach

$X_2(t)$

$X_1(t)$

$X_3(t)$

$s_1(t)$

$s_2(t)$

Chen et al, 2009, Neuroimage

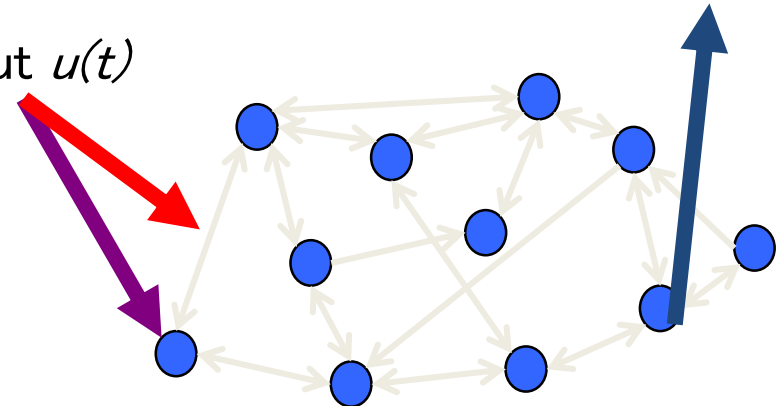| Measure	Immume to Field Spread	Directed	Nonlinear	Direct
Cross-Correlation	Y (l>0)	N	N	N
Coherence	Y (imaginary)	N	N	N
PLV/PLI	Y	N	N	N
Granger (bivariate)	Y	Y	N	N
Mutual Information	N	N	Y	N
Generalised Synchrony	N	Y	Y	N
Transfer Entropy	Y	Y	Y	N
MVAR (eg, PDC)	Y	Y	N	Y
Generative (eg, DCM)	Y	Y	Y	Y

The End

DCM Neural Level

System changes depend on:

- the current state z

- the connectivity θ

- external inputs u
  - driving (to nodes)
  - modulatory (on links)

- time constants & delays
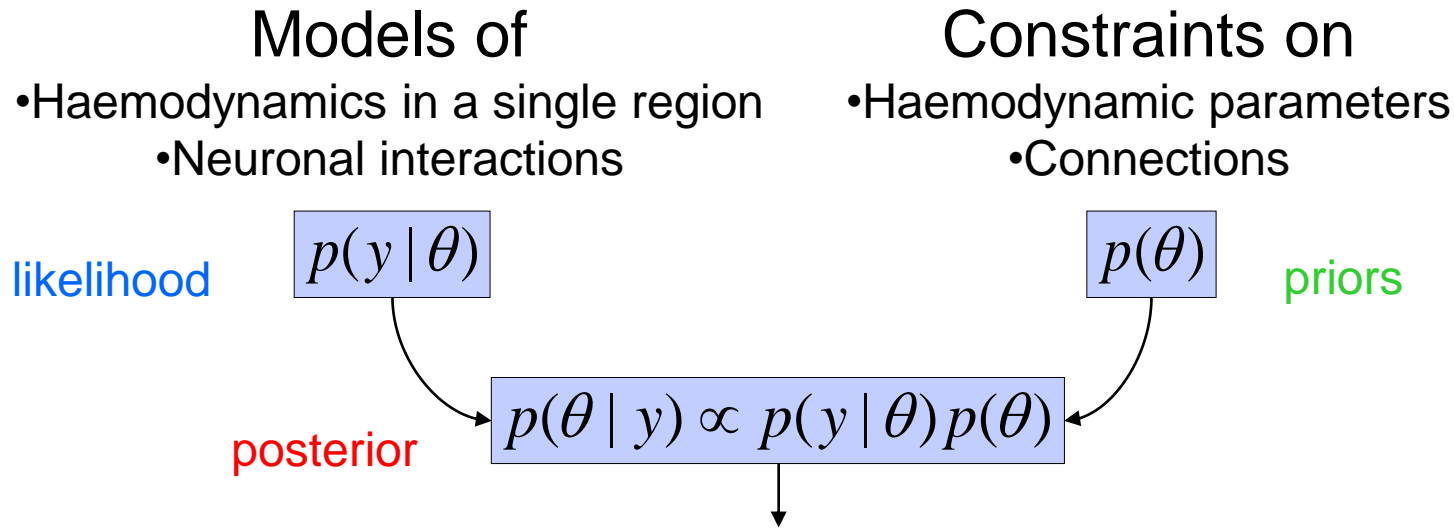
(cf GLM, "inputs" to all nodes simultaneously!)

system $z(t)$ state

Input $u(t)$
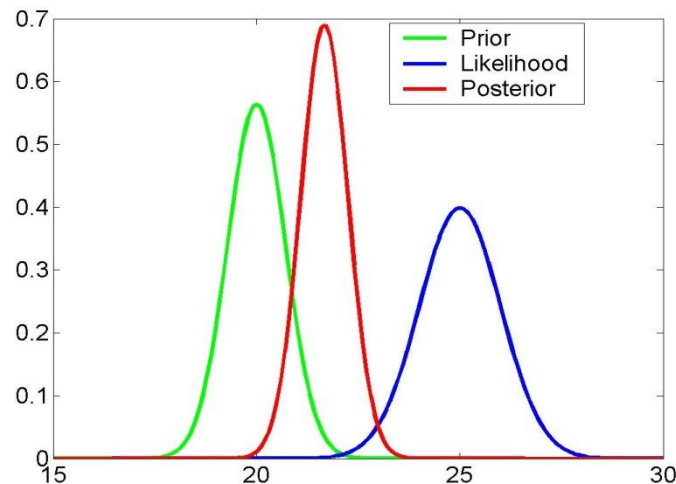
connectivity parameters θ

$$\frac{dz}{dt} = F(z, u, \theta)$$

DCM Estimation: Bayesian framework

Models of
- Haemodynamics in a single region
- Neuronal interactions

Constraints on
- Haemodynamic parameters
- Connections

likelihood $p(y \mid \theta)$

$p(\theta)$ priors

posterior

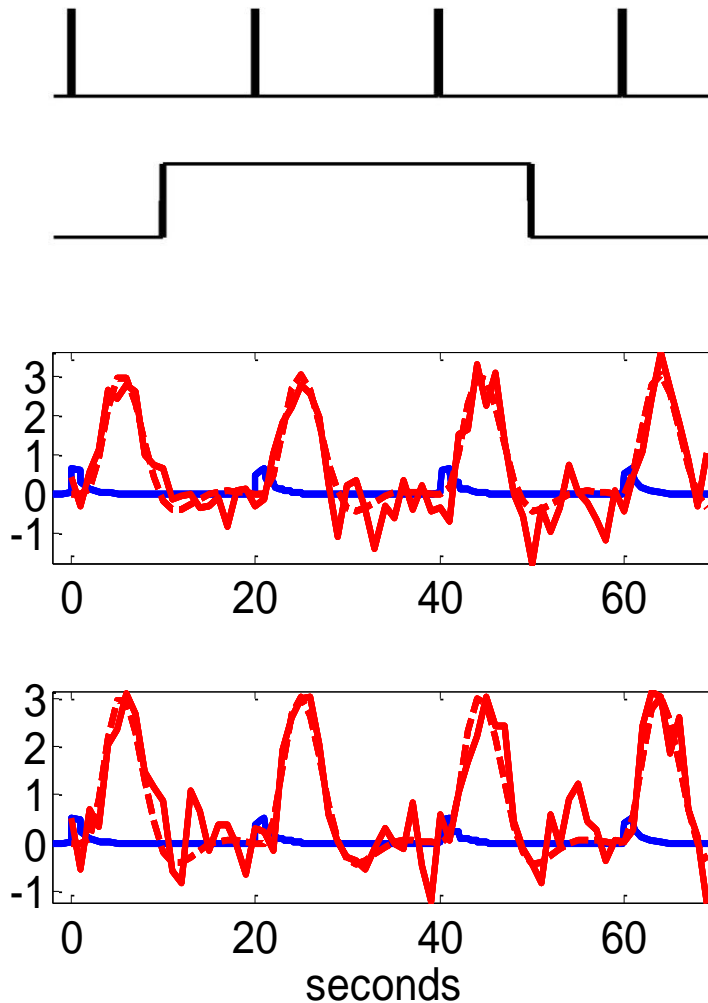$$p(\theta \mid y) \propto p(y \mid \theta) p(\theta)$$
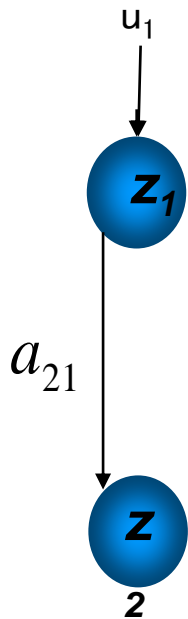
Bayesian estimation

Inferences on:
1. Parameters
2. Models

Parameter estimation: an example

Input coupling, c_1

c_1
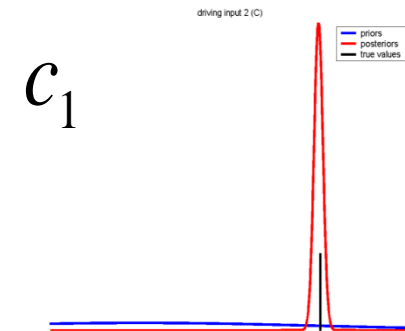
Forward coupling, a_{21}

a_{21}

u_1

z_1

a_{21}

z_2

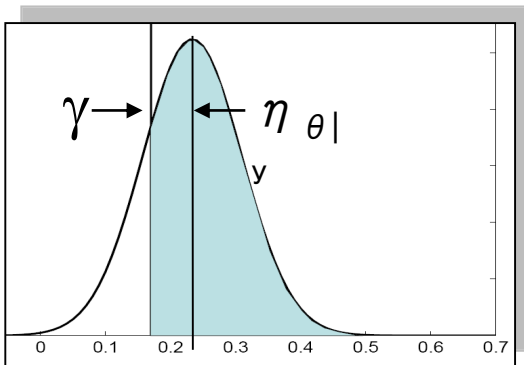seconds
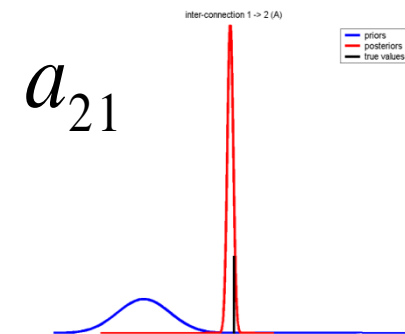
Prior density ——   Posterior density ——   true values ——

Inference about DCM parameters.

Bayesian single subject analysis

- The model parameters are distributions that have a mean $\eta_{\theta|y}$ and covariance $C_{\theta|y}$.

    - Use of the cumulative normal distribution to test the probability that a certain parameter is above a chosen threshold γ :

Classical frequentist test across Ss

- Test summary statistic: mean $\eta_{\theta|y}$

    - One-sample t-test: Parameter>0?

    - Paired t-test:
      parameter 1 > parameter 2?

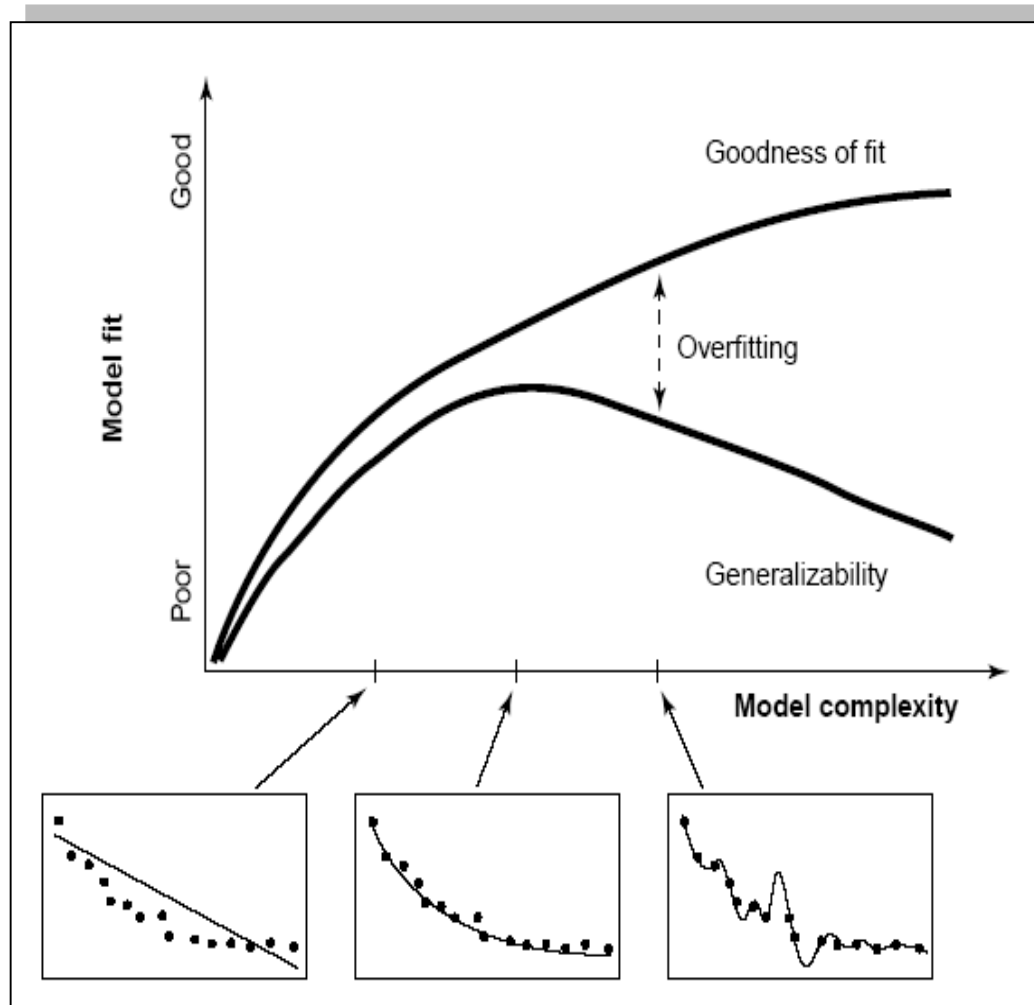   - rmANOVA: e.g. in case of multiple sessions per subject

Model comparison and selection

Given competing hypotheses, which model is the best?

$$\log p(y\,|\,m) =$$
$$accuracy(m) -$$
$$complexity(m)$$

$$B_{ij} = \frac{p(y\,|\,m = i)}{p(y\,|\,m = j)}$$

Bayes Factor

Pitt & Miyung (2002) *TICS*